QUIZ 4 NAME: SOLUTIONS

April 17, 2017

(1) If the derivative of $f(x)$ is $f'(x) = (x-7)(x+1)(x+5)$, where does f have local maxima and local minima?

SOLUTION: $f(x)$ has critical points at $x = 7$, $x = -1$, and $x = -5$. We then need to use some test points to figure out which ones are maxima, which ones are minima, and which ones are neither.

$$f'(-6) = (-13)(-5)(-1) < 0$$
$$f'(-2) = (-9)(-1)(3) > 0$$
$$f'(0) = (-7)(1)(5) < 0$$
$$f'(10) = (3)(11)(15) > 0$$

Hence, the function is decreasing until $x = -5$, after which it is increasing until -1, and then decreasing until 7, and increasing thereafter. So there are local minima at $x = -5$ and $x = 7$, and a local maxima at $x = -1$.

(2) Find $\lim_{t \to 0} \frac{\sin(t) - t}{t^2}$.

SOLUTION: Plugging in $t = 0$, we see that this is of the form $0/0$, so we can use L'Hôpital's rule.

$$\lim_{t \to 0} \frac{\sin(t) - t}{t^2} = \lim_{t \to 0} \frac{\cos(t) - 1}{2t}$$

This is still an indeterminate form $0/0$, so we can apply L'Hopital's rule again.

$$\begin{aligned} \lim_{t \to 0} \frac{\sin(t) - t}{t^2} &= \lim_{t \to 0} \frac{\cos(t) - 1}{2t} \\ &= \lim_{t \to 0} \frac{-\sin(t)}{2} = \boxed{0} \end{aligned}$$