# RELATED RATES PRACTICE

March 22, 2017

**NAME:** SOLUTIONS

STEPS FOR SOLVING PROBLEMS WITH RELATED RATES

(1) Draw a picture representing the problem.

(2) Write down what you know and what you need to find out.

(3) Label your picture with the known and unknown variables.

(4) Find the equation that relates the variables.

(5) Take the derivative of the equation relating the variables with respect to $t$.
Remember your Derivative Rules!

(6) Substitute the given values into the equation from Step 5.

(7) Use the equation from Step 4 to find the missing pieces of information.

(8) Solve for the final missing piece and you're done!

(1) If the radius of a balloon is increasing at a constant rate of 0.03 in/min, how fast is the volume of the balloon changing at the time when its radius is 5 inches?

SOLUTION: We can represent the balloon by a sphere. The volume of a sphere is

$$V = \frac{4}{3}\pi r^3.$$

We are given $\frac{dr}{dt} = 0.03$ in /min and asked to find $\frac{dV}{dt}$ when $r = 5$ inches. To that end, we differentiate the formula for the volume with respect to $t$.

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Then plug in the values $r = 5$ and $\frac{dr}{dt} = 0.03$ to find

$$\frac{dV}{dt} = 4(25)(0.03) = 3 \text{ in}^3/\text{min}$$

(2) An oil spill expands in a circular pattern. Its radius increases at a constant rate of $1 \text{ m/s}$. At time $t = 0$, the radius is 120 meters. What is the rate of change of the area of the spill at time $t = 1$ minute?

SOLUTION: The oil spill is a circle of radius $r$. The relation of the area of this circle and the radius is

$$A = \pi r^2.$$

We are given $\frac{dr}{dt} = 1 \text{ m/s}$ and asked to find $\frac{dA}{dt}$ when $t = 1$ minute.

To relate the radius and time, note that the initial radius is 120 meters, and it expands at one meter every second. So for $t$ measured in seconds, $r(t) = 120 + t$ meters.

To do that, we differentiate both sides of the area equation by $t$ to find

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}.$$

Now plug in $\frac{dr}{dt} = 1$ and $r(t) = 120 + t$. We seek the answer for $t = 1$ minute $= 60$ seconds, so $r(60) = 180$. Therefore,

$$\frac{dA}{dt} = 2\pi(180)(1) = 360\pi \text{ m}^2/\text{s}$$

(3) The length of a rectangle increases by 3 ft/min while the width decreases by 2 ft/min. When the length is 15 ft and the width is 40 ft, what is the rate of change of:

(a) the area?

SOLUTION: The area of a rectangle is given by $A = \ell w$, where $\ell$ is the length and $w$ is the width. We are given $\frac{d\ell}{dt} = 3 \text{ ft/min}$ and $\frac{dw}{dt} = -2 \text{ ft/min}$ and $\ell = 15$ ft and $w = 40$ ft. We are asked to find $\frac{dA}{dt}$. So we differentiate the formula for area:

$$\frac{dA}{dt} = \frac{d\ell}{dt} w + \ell \frac{dw}{dt}$$

Now plug in the given values to find

$$\frac{dA}{dt} = 3(40) + 15(-2) = 90 \text{ ft}^2/\text{min}.$$

(b) the perimeter?

SOLUTION: The perimeter of a rectangle is given by

$$P = 2\ell + 2w,$$

where $\ell$ is the length and $w$ is the width. We are given $\frac{d\ell}{dt} = 3 \text{ ft/min}$ and $\frac{dw}{dt} = -2 \text{ ft/min}$ and $\ell = 15$ ft and $w = 40$ ft. We are asked to find $\frac{dP}{dt}$. So we differentiate the formula for perimeter:

$$\frac{dP}{dt} = 2\frac{d\ell}{dt} + 2\frac{dw}{dt}$$

And then plug in the given values to find

$$\frac{dP}{dt} = 2(3) + 2(-2) = 2 \text{ ft/min}$$

(4) The volume of a tree is given by $V = \frac{1}{12\pi}C^2h$, where $C$ is the circumference of the tree at ground level (in meters) and $h$ is the height of the tree (in meters). Both $C$ and $h$ are functions of time (in years).

(a) Find a formula for $\dfrac{dV}{dt}$. What does it represent?

SOLUTION: Differentiate both sides of the formula for volume with respect to $t$. Don't forget the product rule!

$$\frac{dV}{dt} = \frac{1}{12\pi}\left(2Ch\frac{dC}{dt} + C^2\frac{dh}{dt}\right)$$

This represents the rate of change of volume of the tree, or how fast the tree is growing.

(b) Suppose the circumference grows at a rate of $0.2$ meters per year and the height grows at a rate of $4$ meters per year. How fast is the tree growing when the circumference is $5$ meters and the height is $22$ meters?

SOLUTION: We are given $\frac{dC}{dt} = 0.2$ meters per year and $\frac{dh}{dt} = 4$ meters per year, as well as $C = 5$ meters and $h = 22$ meters. We are asked to find $\frac{dV}{dt}$, which is given as in the formula from part (a). So plug the values in

$$\frac{dV}{dt} = \frac{1}{12\pi}\left(2(5)(22)(0.2) + (5)^2(4)\right) = \frac{1}{12\pi}(44 + 100) = \frac{1}{12\pi}(144) = \frac{12}{\pi} \text{ meters/year.}$$