DEFINITE INTEGRALS   NAME: SOLUTIONS
April 28, 2017

If we find the area under a curve on an interval $[a, b]$ using an infinite number of rectangles with the base length of the rectangles getting closer and closer to zero, we use this notation, where f(x) is the function of the curve:

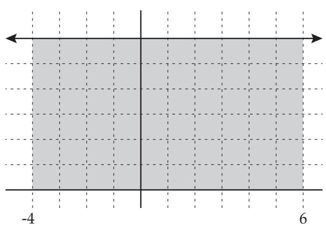
$$\lim_{N\to\infty}\sum_{i=1}^{n} f\left(a+i\frac{(b-a)}{n}\right)\left(\frac{(b-a)}{n}\right)=\int_a^b f(x)dx$$

The limit is called the **definite integral** of f from a to b. The number a is the **lower limit** of integration, and the number b is the **upper limit** of integration.

The first way we are going to evaluate such integrals is to look for common geometric shapes that we already know how to determine the area of (circles, triangles, rectangles, or a combination of those). Try some! To do so, sketch the curve given and then find its area.

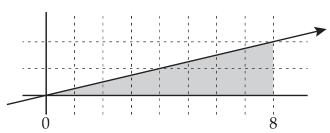
(1) $\int_{-4}^{6} 6\,dx$

SOLUTION: $\int_{-4}^{6} 6\,dx$ $\boxed{=6\cdot 10=60}$ square units. This integral represents the area shaded below, so the area is the area of a (6 by 10) rectangle.



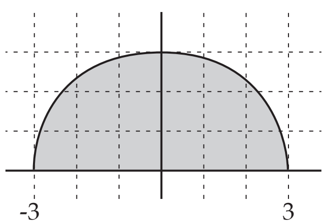
(2) $\int_{0}^{8} \frac{x}{4}\,dx$

SOLUTION: $\int_{0}^{8} \frac{x}{4}\,dx$ $\boxed{=\frac{1}{2}\cdot 8\cdot 2=8}$ square units. This integral represents the area shaded below, so the area is the area of a ($b=8$ by $h=2$) triangle.



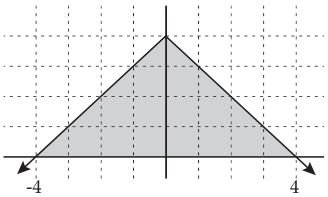
(3) $\int_{-3}^{3} \sqrt{9-x^2}\,dx$

SOLUTION: $\int_{-3}^{3} \sqrt{9-x^2}\,dx$ $\boxed{=\frac{1}{2}\cdot\pi\cdot 3^2=\frac{9\pi}{2}}$ square units. The function $\sqrt{9-x^2}$ is the top half of a circle of radius 3, so the integral represents the area shaded below. This has the area of half of a radius 3 circle.



(4) $\int_{-4}^{4} (4-|x|)\ dx$

SOLUTION: $\int_{-4}^{4} (4-|x|)\ dx \boxed{= \frac{1}{2}\cdot 8\cdot 4 = 16}$ square units. This integral represents the area shaded below, so the area is the area of a ($b = 8$ by $h = 4$) triangle.



Using what you know about area under a curve, try these! Sketches may help you see what is going on in each problem.

(1) Given $\int_{0}^{3} f(x)\ dx = 4$ and $\int_{3}^{6} f(x)\ dx = -1$ evaluate:

(a) $\int_{0}^{6} f(x)\ dx$

SOLUTION: $\int_{0}^{6} f(x)\ dx = \int_{0}^{3} f(x)\ dx + \int_{3}^{6} f(x)\ dx = 4 - 1 = \boxed{3}$.

(b) $\int_{6}^{3} f(x)\ dx$

SOLUTION: $\int_{6}^{3} f(x)\ dx = -\int_{3}^{6} f(x)\ dx = -(-1) = \boxed{1}$.

(c) $\int_{3}^{6} -5f(x)\ dx$

SOLUTION: $\int_{3}^{6} -5f(x)\ dx = -5\int_{3}^{6} f(x)\ dx = -5\cdot(-1) = \boxed{5}$.

(2) Given $\int_{2}^{4} x^3\ dx = 60 \quad \int_{2}^{4} x\ dx = 6 \quad \int_{2}^{4} dx = 2$ evaluate:

(a) $\int_{2}^{2} x^3\ dx$

SOLUTION: $\int_{2}^{2} x^3\ dx = \boxed{0}$ because $\int_{a}^{a} f(x)\ dx = 0$ for any a and any $f(x)$.

(b) $\int_{2}^{4} \left(10 + 4x - 3x^3\right)\ dx$

SOLUTION:

$$\begin{aligned}\int_{2}^{4} \left(10 + 4x - 3x^3\right)\ dx &= 10\int_{2}^{4} 1\ dx + 4\int_{2}^{4} x\ dx - 3\int_{2}^{4} x^3\ dx \\ &= (10\cdot 2) + (4\cdot 6) - (3\cdot 60) = \boxed{136}\end{aligned}$$

This material is based upon original Active Calculus materials produced by the University of Nebraska at Omaha.