SUMMATION NOTATION — NAME: SOLUTIONS

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In calculus, we do a lot of adding. We will introduce two "fancy adding machines" in the next couple of days. The first one uses $\sum$ and is called *Sigma Notation*.

Example:
$$\sum_{n=1}^{5}(2n) = 2(1) + 2(2) + 2(3) + 2(4) + 2(5) = 30$$

Your turn! Find the sum of:
$$\sum_{k=3}^{9}(k^2 + 1)$$

SOLUTION:

$$\begin{aligned}\sum_{k=3}^{9}(k^2+1) &= (3^2+1)+(4^2+1)+(5^2+1)+(6^2+1)+(7^2+1)+(8^2+1)+(9^2+1)\\ &= 10+17+26+37+50+65+82\\ &= 287\end{aligned}$$

We have a few formulæ for sums that show up frequently.

$$\begin{aligned}\sum_{k=1}^{n} k &= \frac{n(n+1)}{2}\\ \sum_{k=1}^{n} k^2 &= \frac{n(n+1)(2n+1)}{6}\\ \sum_{k=1}^{n} k^3 &= \left[\frac{n(n+1)}{2}\right]^2\end{aligned}$$

Use what we know about sums and the above formulæ to evaluate

1. $\displaystyle\sum_{k=1}^{17} (2+k) =$

   SOLUTION:

$$\begin{aligned}\sum_{k=1}^{17} (2+k) &= \sum_{k=1}^{17} 2 + \sum_{k=1}^{17} k\\ &= 17*2+\frac{17(17+1)}{2} = 34+153 = \boxed{187}\end{aligned}$$

2. $\displaystyle\sum_{k=18}^{71} k(k-1) =$

   SOLUTION:

$$\begin{aligned}\sum_{k=18}^{71} k(k-1) &= \sum_{k=18}^{71} k^2-k\\ &= \sum_{k=18}^{71} k^2 - \sum_{k=18}^{71} k\\ &= \left(\sum_{k=1}^{71} k^2 - \sum_{k=1}^{17} k^2\right) - \left(\sum_{k=1}^{71} k - \sum_{k=1}^{17} k\right)\\ &= \left(\frac{71(71+1)(2*71+1)}{6} - \frac{17(17+1)(2*17+1)}{6}\right) - \left(\frac{71(71+1)}{2} - \frac{17(17+1)}{2}\right)\\ &= \boxed{117648}\end{aligned}$$

3. $\displaystyle\sum_{k=1}^{5} \frac{k^3}{225} + \left(\sum_{k=1}^{5} k\right)^3 =$

   SOLUTION:

$$\begin{aligned}\sum_{k=1}^{5} \frac{k^3}{225} + \left(\sum_{k=1}^{5} k\right)^3 &= \frac{1}{225}\sum_{k=1}^{5} k^3 + \left(\frac{5(5+1)}{2}\right)^3\\ &= \frac{1}{225}\left(\frac{5(5+1)}{2}\right)^2 + \left(\frac{5(5+1)}{2}\right)^3\\ &= \frac{1}{225}*15^2+15^3 = 1+3375 = \boxed{3376}\end{aligned}$$