Volumes and Areas

Name: ____________________

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(1) Sketch the region enclosed by the curves and set up an integral to compute it's area, but do not evaluate.

(a) $y = 4 - x^2$, $y = x^2 - 4$

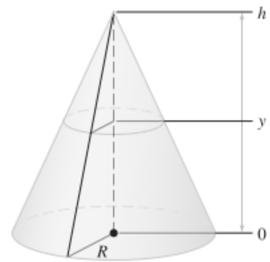
(b) $y = x^2 - 6$, $y = 6 - x^3$, $x = 0$

(c) $y = x\sqrt{x-2}$, $y = -x\sqrt{x-2}$, $x = 4$

(d) $x = 2y$, $x + 1 = (y-1)^2$

(e) $y = \cos x$, $y = \cos(2x)$, $x = 0$, $x = \frac{2\pi}{3}$

(2) Let V be the volume of a right circular cone of height h whose base is a circle of radius R. Find its volume V.

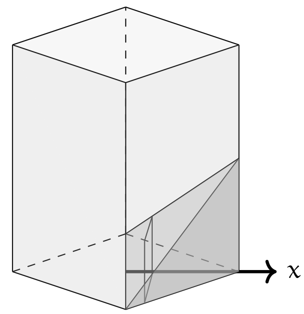


(3) *Going the other way.* Sketch a region whose area is represented by

$$\int_{-\sqrt{2}/2}^{\sqrt{2}/2} (\sqrt{1-x^2} - |x|)\,dx$$

and evaluate it using geometry.

(4) Consider the following box, with square top and bottom lids of side s. A plane inclined at an angle of 45 degrees passes through a diagonal of the bottom lid. Find the volume of the region within the box and below the plane.



(5) Sketch the region enclosed by the curves, and determine the cross section perpendicular to the x-axis. Set up an integral for the volume of revolution obtained by rotating the region around the x-axis, but do not evaluate.

(a) $y = x^2 + 2$, $y = 10 - x^2$.

(b) $y = 16 - x$, $y = 3x + 12$, $x = -1$.

(c) $y = \frac{1}{x}$, $y = \frac{5}{2} - x$.

(d) $y = \sec x$, $y = 0$, $x = -\frac{\pi}{4}$, $x = \frac{\pi}{4}$.

(6) Sketch the solid obtained by rotating the region underneath the graph of f over the interval about the given axis, and calculate its volume using the shell method.

(a) $f(x) = x^3$, $x \in [0, 1]$, about $x = 2$.

(b) $f(x) = x^3$, $x \in [0, 1]$, about $x = -2$.

(c) $f(x) = \frac{1}{\sqrt{x^2+1}}$, $x \in [0, 2]$, about $x = 0$.

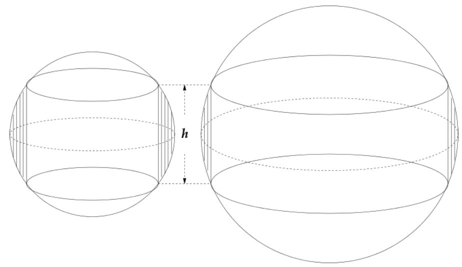
(7) Use the most convenient method (disk/washer or shell) to find the given volume of rotation.

(a) Region between $x = y(5-y)$ and $x = 0$, rotated about the y-axis.

(b) Region between $x = y(5-y)$ and $x = 0$, rotated around the x-axis.

(c) Region between $y = x^2$ and $x = y^2$, rotated about $x = 3$.

(8) *The napkin-ring problem.* Show that when a hole of height h is drilled straight through the center of a sphere, the volume of the remaining band does not depend on the size of the sphere.



This means that a napkin ring of height 1cm built from a marble will have the same volume as one built from the Sun!