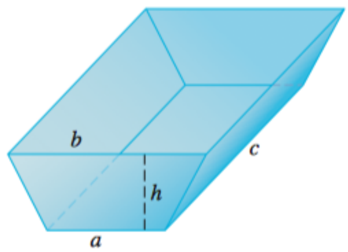
§6.5 (WORK AND ENERGY) NAME: ____________________

10 July 2018

(1) Calculate the work required to lift a 3-meter chain over the side of a building if the chain has variable density $\lambda(x) = x^2 - 3x + 10$ kg/m for $0 \leq x \leq 3$. Assume that the chain is hanging off the edge of the building, with the bottom of the chain at $x = 0$ and the top at $x = 3$.

(2) A 3 meter chain with mass density $\rho(x) = 2x(4 - x)$ kg/m lies on the ground. Calculate the work required to lift the chain from the front end so that its bottom is 2 meters above the ground.

(3) Calculate the work (in Joules) required to pump all of the water out of a trough as in the picture, where the water exits by pouring over the sides. Distances are in meters, and the density of water is 1000 kg/m^3.



§8.1 (INTEGRATION BY PARTS) NAME:________________________

28 July 2018

(1) Evaluate the integral.

(a) $\int xe^{-x}\,dx$

(b) $\int x^3 e^{x^2}\,dx.$

(c) $\int_1^3 \ln x\,dx.$

(d) $\int xe^{2x}\,dx$

(e) $\int x^3 \ln x \, dx$

(f) $\int x \cos 2x \, dx$

(g) $\int \frac{\ln x}{x^2} \, dx$

(h) $\int \frac{\ln(\ln x)}{x} \, dx$

(i) $\displaystyle\int_0^1 \frac{x^3}{\sqrt{9+x^2}}\,dx$

(j) $\displaystyle\int x^4 e^{7x}\,dx$

(k) $\displaystyle\int \frac{(\ln x)^2}{x^2}\,dx$

(2) Find the volume of the solid obtained by revolving $y = \cos x$ for $0 \leq x \leq \pi/2$ around the y-axis.

(3) (a) Derive the reduction formula: $\displaystyle\int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx$

(b) Define functions $P_n(x)$ by the formula $\displaystyle\int x^n e^x \, dx = P_n(x)e^x$. Use the reduction formula from the previous part to prove that $P_n(x) = x^n - nP_{n-1}(x)$.

(c) Use the recursion formula from the previous part to find $P_n(x)$ for $n = 0, 1, 2, 3, 4$.