HOMEWORK QUIZ 8

Math 1910

NAME: SOLUTIONS

26 October 2017

(1) Evaluate the integral: $\int \sin^2(\theta)\cos^2(\theta)\,d\theta$.

SOLUTION: First use the identity $\cos^2\theta = 1 - \sin^2\theta$ to write

$$\int \cos^2\theta \sin^2\theta\,d\theta = \int (1-\sin^2\theta)\sin^2\theta\,d\theta = \int \sin^2\theta\,d\theta - \int \sin^4\theta\,d\theta.$$

Using the reduction formula for $\sin^m(x)$,

$$\begin{aligned}\int \cos^2\theta \sin^2\theta\,d\theta &= \int \sin^2\theta\,d\theta - \left(-\frac{1}{4}\sin^3\theta\cos\theta + \frac{3}{4}\int \sin^2\theta\,d\theta\right)\\ &= \frac{1}{4}\sin^3\theta\cos\theta + \frac{1}{4}\int \sin^2\theta\,d\theta\\ &= \frac{1}{4}\sin^3\theta\cos\theta + \frac{1}{4}\left(-\frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}\int d\theta\right)\\ &= \boxed{\frac{1}{4}\sin^3\theta\cos\theta - \frac{1}{8}\sin\theta\cos\theta + \frac{1}{8}\theta + C}\end{aligned}$$

(2) Evaluate the integral: $\int_1^2 x \ln(x)\, dx$.

SOLUTION: Let $u = \ln(x)$, $dv = x\, dx$. Then $du = \frac{1}{x}\, dx$ and $v = x^2/2$.

$$
\begin{aligned}
\int_1^2 x \ln(x)\, dx &= \frac{1}{2}x^2 \ln(x)\Big|_1^2 - \int_1^2 \frac{x^2}{2}\frac{1}{x}\, dx \\
&= \frac{1}{2}x^2 \ln(x)\Big|_1^2 - \int_1^2 \frac{x}{2}\, dx \\
&= \frac{1}{2}\left(4\ln(4) - \ln(1)\right) - \frac{1}{4}x^2\Big|_1^2 \\
&= 2\ln(4) - \frac{1}{4}(4-1) \\
&= \boxed{2\ln(4) - \frac{3}{4}}
\end{aligned}
$$

§8.2 (TRIG INTEGRALS) NAME: ______________________

11 July 2018

(1) Evaluate the integral.

(a) $\int \cos(x)\sin^5(x)\,dx$

SOLUTION: Substitute $u = \sin x$, $du = \cos(x)\,dx$.

$$\int \cos(x)\sin^5(x)\,dx = \int u^5\,du = \frac{u^6}{6} + C = \boxed{\frac{\sin^6(x)}{6} + C.}$$

(b) $\int \tan(x)\,dx$

SOLUTION: Rewrite $\tan(x) = \frac{\sin(x)}{\cos(x)}$ and substitute $u = \cos(x)$. The answer is

$$\boxed{\int \tan(x)\,dx = \ln|\sec(x)| + C.}$$

(c) $\int \cos^2(4x)\,dx$

SOLUTION: Use the substitution $u = 4x$ and $du = 4\,dx$. Then

$$\begin{aligned}\int \cos^2(4x)\,dx &= \frac{1}{4}\int \cos^2(u)\,du\\ &= \frac{1}{4}\left(\frac{1}{2}u + \frac{1}{2}\sin(u)\cos(u)\right) + C\\ &= \boxed{\frac{1}{2}x + \frac{1}{8}\sin(4x)\cos(4x) + C}\end{aligned}$$

(d) $\int \tan^3(x)\sec(x)\,dx$

SOLUTION: Use the identity $\tan^2(x) = \sec^2(x) - 1$ to rewrite the integral

$$\int \tan^3(x)\sec(x)\,dx = \int \tan(x)(\sec^2(x) - 1)\sec(x)\,dx$$

Then substitute $u = \sec(x)$, $du = \sec(x)\tan(x)\,dx$. The answer is

$$\boxed{\frac{1}{3}\sec^3(x) - \sec(x) + C.}$$

(e) $\int \sin^3(x)\cos^3(x)\,dx$

SOLUTION: Rewrite $\sin^3(x) = (1 - \cos^2(x))\sin(x)$, and let $u = \cos(x)$.

(f) $\int x\sec^2(x)\,dx$

SOLUTION: Use integration by parts, with $u = x$ and $dv = \sec^2(x)\,dx$.

(g) $\int \sin^4(x)\cos^2(x)\,dx$

SOLUTION:

$$\int \sin^4(x)\cos^2(x)\,dx = \int \sin^4(x)(1 - \sin^2(x))\,dx = \int \sin^4(x)\,dx - \int \sin^6(x)\,dx$$

Using the reduction formula, we get

$$\int \sin^4(x)\cos^2(x)\,dx = \frac{1}{6}\sin^5(x)\cos(x) - \frac{1}{24}\sin^3(x)\cos(x) - \frac{1}{16}\sin(x)\cos(x) + \frac{1}{16}x + C$$

(h) $\displaystyle\int \frac{\cos^5(x)}{\sin^3(x)}\,dx$

SOLUTION: Using the identity $\cos^2(x) = 1\sin^2(x)$, we have

$$\int \frac{\cos^5(x)}{\sin^3(x)}\,dx = \int \frac{(1-\sin^2(x))^2}{\sin^2(x)}\cos(x)\,dx$$

Then substitute $u = \sin(x)$. The answer is

$$\boxed{\frac{-1}{2\sin^2(x)} - 2\ln(\sin(x)) + \frac{1}{2}\sin^2(x) + C.}$$

(i) $\displaystyle\int_0^{\pi} \sin(2x)\sin(x)\,dx$

ANSWER: $\displaystyle\int_0^{\pi} \sin(2x)\sin(x)\,dx = 0$