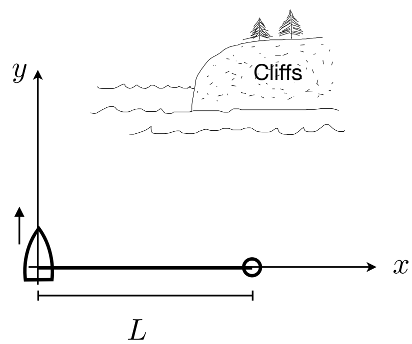
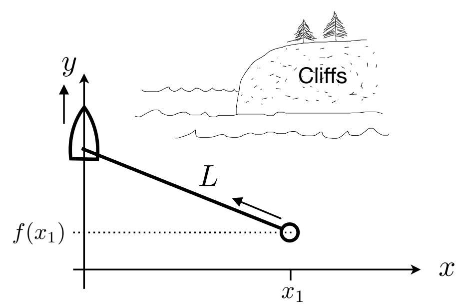
Introduction: This workshop gives a glimpse of how calculus pops up in all sorts of unexpected places. Here we travel back in time to the carefree days of summer. Imagine: you're at a summer party hosted by friends at their lake house, and a group of you decide to take the motorboat and go tubing. Fatima claims the tube first and swims off to the end of the rope while you all get settled on the boat, and then off you go! You're cruising along when you notice that Pablo is driving the boat rather close to an approaching cliff face.





Will Fatima have to let go of the rope or will she make it past the rocky cliffs safely? To find out, we will look for the path $y = f(x)$ that the inner tube traces out as it follows the motorboat. The boat is assumed to travel along the y–axis, as sketched above.

Goals:

- Translate a physical situation into a calculus framework.
- Evaluate an integral of a slope to find the underlying function.

Problems:

a) Use the assumptions:

- $y = f(x)$ defines the path that the *inner tube* traces over the water;
- the rope remains taut throughout the motion, with constant length L;
- the motion of the inner tube is always in the direction of the rope, i.e. at any given moment, the rope is tangential to the tube's path $y = f(x)$;

to find an expression for $\frac{dy}{dx} = f'(x)$. (*Hint:* Start with a rough sketch of $y = f(x)$ and then think about the meaning of "tangential".)

b) Integrate your $f'(x)$ from part **a)** to find the path $y = f(x)$ that the inner tube will follow. Be sure to use the initial conditions to solve for the constant of integration. (*Hint:* Use the identity $\sin^2\theta = 1 - \cos^2\theta$ to simplify the integrand.)

c) Given the path found in part **b)**, and some curve $y = g(x)$ that describes the cliff edge, briefly explain how you would find out if Fatima makes it past the cliffs safely.

d) Are the assumptions in part **a)** reasonable? What physical effects do we ignore? Does the speed of the boat affect our model?

§9.1 (ARC LENGTH & SURFACE AREA) NAME: ______________________________

17 July 2018

The **arc length** of the graph of $y = f(x)$ over the interval $[a, b]$ is

$$\int_a^b \sqrt{1 + f'(x)^2}\, dx.$$

The **surface area** of the solid of revolution S formed by rotating the graph of $y = f(x)$ about the x-axis over the interval $[a, b]$ is

$$\int_a^b 2\pi|f(x)|\sqrt{1 + f'(x)^2}\, dx.$$

Compute the surface area of revolution about the x-axis over the given interval.

(1) $y = (4 - x^{2/3})^{3/2}, \ [0, 8]$

(2) $y = e^{-x}, \ [0, 1]$

(3) $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, \quad [1, e]$

(4) $y = \cos(x), \; [0, \pi]$