§8.9 (Numerical Integration) Name: ______________________

20 July 2018

(1) Find the T_4 approximation for $\int_0^4 \sqrt{x}\, dx$.

(2) State whether M_{10} underestimates or overestimates $\int_1^4 \ln(x)\, dx$.

(3) Approximate the arc length of the curve $y = \sin(x)$ over the interval $[0, \pi/2]$ using the midpoint approximation M_8.

(4) Find a number N for which $\text{Error}(T_N) \leq 10^{-6}$ for $\int_0^3 e^{-x}\, dx$.

(5) Since Simpson's Rule can be derived by using quadratic polynomials (parabolas) to approximate a function, it makes sense that Simpson's rule gives the exact value for integrals of quadratic polynomials.

(a) Prove the statement above. In other words, show that the integral of a quadratic polynomial $f(x) = A + Bx + Cx^2$ over an interval $[a, b]$ exactly coincides with the Simpson's Rule approximation S_2.

(b) Perhaps unexpectedly, Simpson's Rule also gives the exact result for integrals of cubic polynomials. Show this as well: the integral of $g(x) = A + Bx + Cx^2 + Dx^3$ over $[a, b]$ is equal to the Simpson's Rule approximation S_2.

(c) Take another look at the error bound for Simpson's Rule. Is there a quicker way to prove the previous two results without calculating the integrals?