§11.2 (SERIES)
§11.3 (SERIES WITH POSITIVE TERMS)   NAME: ________________________
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CONVERGENCE TESTS FOR SERIES

- **The divergence test:** If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.
- A series that looks like $a_n = cr^n$ is called **geometric.**
  (a) If $|r| \geq 1$, then it diverges.
  (b) If $|r| < 1$, then $\sum_{n=K}^{\infty} cr^n = \frac{cr^K}{1-r}$
- **The integral test:** Assume that $a_n = f(n)$ for $n \geq M$.
  (a) If $\int_M^{\infty} f(x)\,dx$ converges, then $\sum_{n=0}^{\infty} a_n$ converges.
  (b) If $\int_M^{\infty} f(x)\,dx$ diverges, then $\sum_{n=0}^{\infty} a_n$ diverges.
- **The comparison test:**
  (a) If $a_n \leq b_n$, and $\sum_{n=0}^{\infty} b_n$ converges, then $\sum_{n=0}^{\infty} a_n$ converges.
  (b) If $\sum_{n=0}^{\infty} a_n$ diverges, then $\sum_{n=0}^{\infty} b_n$ diverges.
- **Limit comparison test:** Let $\{a_n\}$ and $\{b_n\}$ be sequences with positive terms. Let $L = \lim_{n\to\infty} \frac{a_n}{b_n}$.
  (a) If ______ (1), then $\sum a_n$ converges if and only if $\sum b_n$ converges.
  (b) If ______ (2) and $\sum a_n$ converges, then $\sum b_n$ converges.
  (c) If ______ (3) and $\sum b_n$ converges, then $\sum a_n$ converges.

PROBLEMS

(1) Determine the limit of the series or show that the series diverges.

(a) $\sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$

(b) $\sum_{n=0}^{\infty} e^n$

(c) $\sum_{n=1}^{\infty} \frac{1}{n}$.

(d) $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

(f) $\displaystyle\sum_{n=0}^{\infty} \frac{9^n + 2^n}{5^n}$

(g) $\displaystyle\sum_{n=1}^{\infty} \cos(\pi n)$

(h) $\displaystyle\sum_{n=1}^{\infty} \cos \frac{1}{n}$

(i) $\sum_{n=2}^{\infty} \frac{n^2}{n^4 - 1}$ (Limit Comparison Test)

(j) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + 2^n}$ (Comparison Test)

(k) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ (Integral Test)

(2) Give a counterexample to show that each of the following statements is false.

(a) If the general term a_n tends to zero, then $\sum a_n$ converges.

(b) The Nth partial sum of the infinite series defined by $\{a_n\}$ is equal to a_N.

(c) If $a_n \to L$, then $\sum_{n=0}^{\infty} a_n = L$.

(3) Determine a reduced fraction that is equal to 0.217217217217....

(4) Let $b_n = \frac{\sqrt[n]{n!}}{n}$.

(a) Show that $\ln b_n = \frac{1}{n} \sum_{k=1}^{n} \ln \frac{k}{n}$.

(b) Show that $\ln b_n$ converges to $\int_0^1 \ln x \, dx$. Use this to compute $\lim b_n$.