§11.4 (Alternating Series) Name: ____________________

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Absolute and Conditional Convergence

- **Absolute Convergence:** A series $\sum_{n=1}^{\infty} a_n$ **converges absolutely** if $\sum_{n=1}^{\infty} |a_n|$ converges.
- **Absolute Convergence Theorem:** If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
- **Conditional Convergence:** A series $\sum_{n=1}^{\infty} a_n$ **converges conditionally** if $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges.
- **Alternating Series Test:** If the sequence $\{b_n\}$ is positive and decreasing, and $\lim_{n\to\infty} b_n = 0$, then $S = \sum_{n=1}^{\infty} (-1)^n b_n$ converges. Furthermore, the partial sums satisfy $|S - S_N| < b_{N+1}$.

Problems

(1) Show that $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+1}$ converges conditionally.

(2) Does $\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{n^3+1}$ converges absolutely, conditionally, or not at all?

(3) Consider the series $\sum_{n=2}^{\infty} \frac{\cos n\pi}{(\ln n)^2}$.

(a) Show that the series doesn't converge absolutely by using the Direct Comparison Test.

(b) Does it converge conditionally?

(4) Find a value of N such that the N-th partial sum S_N approximates the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+2)(n+3)}$ with an error of at most 10^{-5} (calculator needed).

(5) Prove that $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$ and $\lim_{n\to 0} \sqrt[n]{1+n} = e$.