§11.6 (Power Series) Name: ____________________

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Power Series

(1) An infinite series of the form $F(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$ is called a **power series** and c is called the **center**.

(2) The **radius of convergence** of $F(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$ is a constant R such that $F(x)$ converges absolutely for $|x-c| < R$ and diverges for $|x-c| > R$. If $F(x)$ converges for all x, then $R = \infty$.

(3) To determine R, use ______ (1)

(4) $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, with radius of convergence $R =$ ______ (2).

(5) If $R > 0$, then a power series $F(x)$ is differentiable on $(c-R, c+R)$, and

$$F'(x) = \sum_{n=1}^{\infty} n a_n (x-c)^{n-1}.$$

$$\int F(x)\,dx = C + \sum_{n=0}^{\infty} \frac{a_n}{n+1}(x-c)^{n+1}.$$

Problems

(1) Show that all three of the following power series have the same radius of convergence, but different behavior at the endpoints.

(a) $\sum_{n=1}^{\infty} \frac{(x-5)^n}{9^n}$

(b) $\sum_{n=1}^{\infty} \frac{(x-5)^n}{n 9^n}$

(c) $\sum_{n=1}^{\infty} \frac{(x-5)^n}{n^2 9^n}$

(2) Use the geometric series formula to expand the function $\frac{1}{1+3x}$ in a power series with center $c = 0$ and determine radius of convergence.

(3) Find a power series expansion for $\ln(1 + x)$ and the interval on which this expansion is valid.