READING ASSIGNMENT 01

§5.2 (Definite Integrals)

NAME: SOLUTIONS

Due 26 June 2018

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- use geometry to compute simple definite integrals;
- write down integrals for the (signed) area under a curve;
- estimate an integral using right endpoint or left endpoint approximations.

REVIEW

- To review summation notation, read pages 227-228 in section 5.1 in the textbook, or watch this YouTube video: https://youtu.be/54Q0KXX_vIs. Another helpful resource is the website at the URL below.

  http://www.columbia.edu/itc/sipa/math/summation.html

- To review functions and graphing, read sections 1.1-1.4 in the textbook. Desmos is a good online tool for visualizing graphs: https://www.desmos.com/calculator.

READING

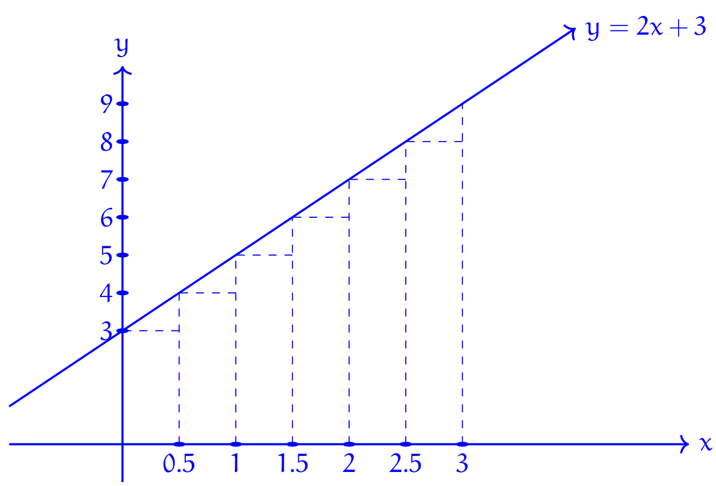
- Read section 5.2 in the textbook, or watch the YouTube video at the URL below and answer the following questions.

  https://www.youtube.com/watch?v=UG3GchWca7c&feature=youtu.be

QUESTIONS

(1) Using at least six rectangles, estimate the area under the graph of $f(x) = 2x + 3$ over the interval $[0, 3]$.

SOLUTION: First, draw a picture:



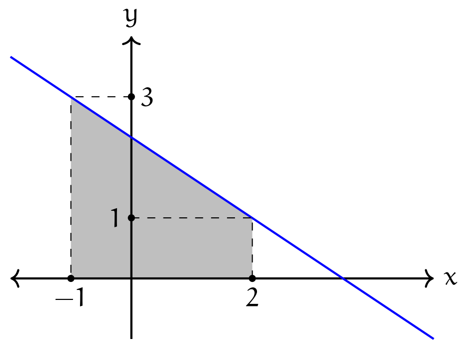
The x-coordinates in question go from $x_0 = 0$ to $x_5 = 3$ by increments of $\Delta x = \frac{1}{2}$. So $x_i = \frac{1}{2} \cdot i = i/2$. The height of the i-th rectangle is $f(x_i) = 2x_i + 3$. Hence, the area of each rectangle is $\frac{1}{2}(2x_i + 3)$. The total area of all rectangles is the sum of the areas of the individual rectangles.

$$\sum_{i=0}^{5}(2x_i + 3)\Delta x = \sum_{i=0}^{5}(2(i/2) + 3)\Delta x = \sum_{i=0}^{5}\frac{i+3}{2} = \frac{3}{2} + \frac{3.5}{2} + \frac{4}{2} + \frac{4.5}{2} + \frac{5}{2} + \frac{5.5}{2} = \frac{25.5}{2} = 12.75.$$

(I chose to use left-hand endpoints in my solution; you may have choosen to use right-hand endpoints.)

(2) Write down integrals to represent the following areas:

(a) the shaded quadrilateral pictured below.



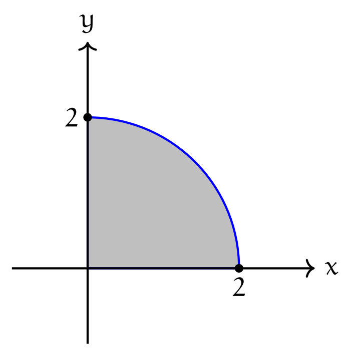
SOLUTION: First, we need to find the equation for the line in the picture. It passes through the points $(-1, 3)$ and $(2, 1)$, and therefore has a slope of $(1-3)(2-(-1)) = -2/3$. So using point-slope form, the equation is

$$y - 1 = \frac{-2}{3}(x-2).$$

Solving for y, we have $y = (-2/3)x - (1/3)$. The area is from $x = -1$ to $x = 2$, so the integral is

$$\boxed{\int_{-1}^{2} -\tfrac{2}{3}x - \tfrac{1}{3}\,dx}$$

(b) the area under the quarter-circle pictured below.



SOLUTION: The equation for a circle is $x^2 + y^2 = r^2$, where r is the radius. This circle has radius 2, so the equation is $x^2 + y^2 = 4$. However, we need to first solve for y to take the integral. We can do that as below:

$$\begin{aligned} x^2 + y^2 &= 4 \\ y^2 &= 4 - x^2 \\ y &= \pm\sqrt{4 - x^2} \end{aligned}$$

Don't forget that when you take a square root, you could get either the plus or minus square root! In this case, because the quarter circle is above the x-axis, we take the positive square root. Hence, the integral is

$$\boxed{\int_{0}^{2} \sqrt{4 - x^2}\,dx}$$