READING ASSIGNMENT 02

NAME: SOLUTIONS

§5.4, §5.5 (Fundamental Theorem of Calculus)

Due 27 June 2018

LEARNING OBJECTIVES

By the end of this lesson, you will be able to:

- apply part 1 of the Fundamental Theorem of Calculus to calculate simple definite integrals,
- apply part 2 of the Fundamental Theorem of Calculus to find derivatives of functions defined as integrals.

REVIEW

- Review the chain rule from section 3.7 in the textbook.

READING

- Read section 5.4, but skip the proof starting below the "Theorem 1" box on page 254 through the "Conceptual Insight" box on page 255.
- Read section 5.5, but skip the proof starting below the "Theorem 1" box on page 259 through the sentence after equation (2) midway through page 260.

QUESTIONS

(1) Assume that $h(x) = f(g(x))$, where both f and g are differentiable functions. If $g(-1) = 2$, $g'(-1) = 3$, $f(2) = -1$ and $f'(2) = 4$, what is the value of $h'(-1)$?

SOLUTION: This is a chain rule question. We can differentiate:

$$h'(x) = f'(g(x)) \cdot g'(x).$$

Then plug in $x = -1$. Notice that there is extraneous information in the question!

$$h'(-1) = f'(g(-1)) \cdot g'(-1) = f'(2) \cdot 3 = -1 \cdot 3 = -3.$$

(2) Answer the following questions clearly, so your classmates could understand.

(a) In your own words, what does the Fundamental Theorem of Calculus say about the relationship between indefinite integrals and definite integrals?

SOLUTION: Answers may vary. One possible answer is the following: An indefinite integral of a function $f(x)$ is a general antiderivative $F(x) + C$ for that function. By the fundamental theorem of calculus, part 1, we may compute a definite integral $\int_a^b f(x)\, dx$ by using this antiderivative (indefinite integral) as

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

So an indefinite integral $\int f(x)\, dx = F(x) + C$ is a template into which we may input bounds; in doing so we learn the area under the graph of $f(x)$ between the bounds we plug in.

(b) Does every continuous function have an antiderivative?

SOLUTION: Yes. If g is a continuous function, then we may define the function

$$G(x) = \int_0^x g(t)\, dt$$

such that

$$\frac{d}{dx} G(x) = \frac{d}{dx} \int_0^x g(t)\, dt = g(x).$$

So $G(x)$ is an antiderivative for $g(x)$.