Reading Assignment 05

§6.3 (Volumes of Revolution), §6.4 (Cylindrical Shells)

LEARNING OBJECTIVES

By the end of this lesson, you will be able to:

• Calculate a volume of a solid of revolution using either the washer method or the shell method.

REVIEW

• This section relies on basic geometry: the Pythagorean theorem, similar triangles, and your ability to visualize the cross-sections of shapes in 3D. You may not need to review these skills, but it will help to keep them in mind!

Reading

- Read section 6.3.
- Read section 6.4.

QUESTIONS

(1) What is the volume of a hollow cylinder of inner radius r, outer radius R, and height h?



ANSWER:

$$\pi(R^2-r^2)h=\pi h(R+r)\Delta r,$$

where $\Delta \mathbf{r} = (\mathbf{R} - \mathbf{r})$.

(2) When should you use the shell method, and when should you use the disk/washer method? List at least two advantages and disadvantages of each.

ANSWER: For the shell method, the shell height is parallel to the axis of rotation, and for the disk/washer method, the disk radius is perpendicular to the axis of rotation. Use the shell method when it's easier to find the shell height, and use the disk method when it's easier to find disk radius.

I think the disk method formula is easier to remember, but that the shell method is by far easier to compute because it doesn't have an $f(x)^2$ in the integrand. The shell method is often also setup for substitution.

When you are revolving around the y-axis, the shell method is still in terms of dx, while the disk method is in terms of dy.

When you are revolving around the x-axis, the disk method is in terms of dx, while the shell method is in terms of dy.

(3) Give an example of a problem that is easier to solve with the shell method, and an example of a problem that is easier to solve with the disk/washer method.

ANSWER: A problem easier to use the shell method for is the following:

• Find the volume V of the solid obtained by rotating the region under the graph of $f(x) = 1 - 2x + 3x^2 - 2x^3$ over [0, 1] around the y-axis.

A problem easier to use the disk method for is the following:

• Find the volume obtained by rotating the graphs of $f(x) = 9 - x^2$ and y = 12 for $0 \le x \le 3$ about the line y = 12.