READING ASSIGNMENT 07

NAME: SOLUTIONS

§8.1 (Integration by parts), §8.2 (Trig integrals)

Due 11 July 2018

LEARNING OBJECTIVES

By the end of this lesson, you will be able to:

- Use integration by parts to evaluate integrals of products.
- Evaluate integrals of the form $\int \sin^n(x)\cos^m(x)\,dx$, and similar integrals involving other trigonometric functions.

REVIEW

- Review trigonometric identities. A good resource is here:

http://www2.clarku.edu/~djoyce/trig/identities.html

READING

- Read section 8.1
- Read section 8.2

QUESTIONS

(1) How do you evaluate an integral like $\int e^x \cos(x)\,dx$ where integrating by parts takes you in a circle?

SOLUTION: If integration by parts takes you in a circle, you can collect like terms and divide by a constant to get what you want. Essentially, set

$$I = \int e^x \cos(x)\,dx$$

and then solve for I.

(2) Which trigonometric identity is used to evaluate $\int \sin^2(\theta)\,d\theta$?

SOLUTION: The power-reducing identity

$$\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}.$$

(3) Describe strategies to integrate $\int \sin^n(x)\cos^m(x)\,dx$ when:

(a) m and n are both even.

SOLUTION: Use the power-reducing identity on either $\sin^n(x)$ or $\cos^m(x)$ to get an integral with either all sines or all cosines. Then repeat: use the power-reducing identity on the other until you have an integral of a sum of either all sines or all cosines with no powers. This integral can be evaluated directly.

(b) m is even and n is odd.

SOLUTION: Substitute $\sin^2(x) = (1-\cos^2(x))$ for all but one of the factors of $\sin(x)$ in the integral. Then use the substitution $t = \cos(x)$, $dt = -\sin(x)\,dx$.

(c) m and n are both odd.

SOLUTION: Same as the previous part.