Reading Assignment 08

Name: Solutions

§8.3 (Trig substitution), §8.5 (Partial fractions)

Due 12 July 2018

Learning Objectives

By the end of this lesson, you will be able to:

- Compute integrals of the form $\int (ax^2 + bx + c)^{n/2}\, dx$ using trigonometric substitution.
- Compute integrals of rational functions using partial fractions.

Review

- Review completing the square and the definitions of sine, cosine, and tangent (i.e. $\sin(x) =$opposite/hypotenuse, etc.).
- Review polynomial long division.

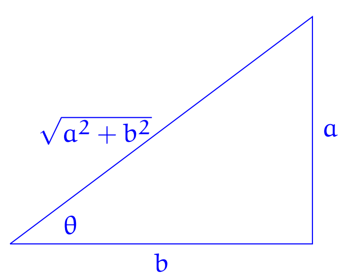
Reading

- Read section 8.3
- Read section 8.5

Questions

(1) If $\tan^{-1}(\theta) = \frac{a}{b}$, then what is $\sin(\theta)$?

Solution: If $\tan^{-1}(\theta) = \frac{a}{b}$, then draw a triangle and use $\tan(\theta) = \text{opposite}/\text{adjacent}$.



From the picture, we see that

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{\sqrt{a^2+b^2}}.$$

(2) Describe the strategy used to integrate $\int \frac{P(x)}{Q(x)}\,dx$ when:

(a) The degree of $P(x)$ is larger than the degree of $Q(x)$.

SOLUTION: Perform polynomial long division to obtain a polynomial function plus a rational function whose denominator has higher degree. Then perform partial fractions on the rational part.

(b) The degree of $Q(x)$ is larger than the degree of $P(x)$, and $Q(x)$ splits into distinct factors of the form $(x-a)$.

SOLUTION: Rewrite

$$\frac{P(x)}{Q(x)} = \frac{P(x)}{(x-a_1)(x-a_2)\cdots(x-a_n)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \ldots + \frac{A_n}{x-a_n}$$

using partial fractions. Each term $(x-a)$ contributes a factor of the form

$$\frac{A}{x-a}.$$

(c) The degree of $Q(x)$ is larger than the degree of $P(x)$, and $Q(x)$ has an irreducible quadratic factor (x^2+a).

SOLUTION: As before, but each factor (x^2+a) in the denominator contributes a factor of the form

$$\frac{Ax+B}{x^2+a}.$$