READING ASSIGNMENT 10

§8.7 (Improper Integrals)

NAME: SOLUTIONS

Due 18 July 2018

LEARNING OBJECTIVES

By the end of this lesson, you will be able to:

- Compute an improper integral as a limit of proper integrals.
- Use the comparison test to determine whether an improper integral converges or diverges.

REVIEW

- Review L'Hôpital's rule: if $\frac{f(x)}{g(x)}$ is one of the indeterminate forms $\frac{0}{0}, \frac{\infty}{\infty}, \ldots$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}.$$

READING

- Read section 8.7

QUESTIONS

(1) Use the definition of improper integrals to rewrite the improper integrals below as limits of proper integrals.

(a) $\displaystyle\int_{-\infty}^{-1} \frac{1}{x}\, dx$

SOLUTION:

$$\lim_{a \to -\infty} \int_{a}^{-1} \frac{1}{x}\, dx$$

(b) $\displaystyle\int_{0}^{2} \frac{1}{(x-1)}\, dx$

SOLUTION:

$$\int_{0}^{2} \frac{1}{(x-1)}\, dx = \lim_{a \to 1} \int_{0}^{a} \frac{1}{x-1}\, dx + \lim_{b \to 1} \int_{b}^{2} \frac{1}{x-1}\, dx$$

(2) When using the comparison test to determine convergence or divergence of improper integrals, we almost always compare to a particular class of improper integral called the p-integrals. For each of the improper integrals below, which p-integral should we compare it to in order to determine convergence or divergence?

(a) $\int_1^\infty \frac{1}{\sqrt{x^3+1}}\,dx$

SOLUTION:

$$\int_1^\infty \frac{1}{x^{3/2}}\,dx$$

(b) $\int_0^{\frac{1}{2}} \frac{1}{x^8+x^2}\,dx$

SOLUTION:

$$\int_0^{\frac{1}{2}} \frac{1}{x^2}\,dx$$