Reading Assignment 13

§11.3 (Series with Positive Terms)

Name: Solutions

Due 23 July 2018

Learning Objectives

By the end of this lesson, you will be able to:

- determine convergence or divergence of series with positive terms by direct comparison, limit comparison, or the integral test,
- recite a proof that the harmonic series diverges using the integral test.

Review

- Review limits, sigma notation, and p-integrals and the integral comparison test from §8.7 (Improper Integrals).

Reading

- Read section 11.3. You may skip the proofs of the theorems.

Questions

(1) Fill in the blanks in the statement of the limit comparison test.

Limit Comparison Test. Let $\{a_n\}$ and $\{b_n\}$ be $\boxed{\text{positive}}^{(1)}$ sequence such that

$$L = \lim_{n\to\infty} \frac{a_n}{b_n}$$

exists. Then

- If $L > 0$, then $\boxed{\sum_{n=1}^{\infty} a_n \text{ converges if and only if } \sum_{n=1}^{\infty} b_n \text{ converges}}^{(2)}$.
- If $\boxed{L = \infty}^{(3)}$ and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ converges as well.
- If $\boxed{L = 0}^{(4)}$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges as well.

(2) Anne is trying to determine convergence of $\sum_{n=1}^{\infty} \frac{e^{-n}}{n}$ below. Grade her work.

$$e^{-n} = \frac{1}{e^{n}} < 1$$

So use the comparison test.

$$\frac{e^{-n}}{n} < \frac{1}{n}, \quad \sum_{n=1}^{\infty} \frac{e^{-n}}{n} < \sum_{n=1}^{\infty} \frac{1}{n}$$

Harmonic Series

diverges

So $\sum_{n=1}^{\infty} \frac{e^{-n}}{n}$ diverges.

SOLUTION: Anne's answer is incorrect because she used the comparison test in the wrong way: the harmonic series diverges, but that doesn't mean that a smaller series diverges as well.

(3) Carefully write a solution to the problem you just graded.

SOLUTION: Consider instead

$$\frac{e^{-n}}{n} = \frac{1}{ne^{n}} < \frac{1}{e^{n}}$$

Then we may compare it to the series

$$\sum_{n=1}^{\infty} \frac{1}{e^{n}}.$$

This series is geometric, and converges. We have

$$\sum_{n=1}^{\infty} \frac{e^{-n}}{n} < \sum_{n=1}^{\infty} \frac{1}{e^{n}},$$

so this series converges as well.