READING ASSIGNMENT 16

§11.6 (Power Series), §11.7 (Taylor Series)

NAME: SOLUTIONS

Due 31 July 2018

LEARNING OBJECTIVES

By the end of this lesson, you will be able to:

- find the radius of convergence of a power series,
- find Taylor series for functions by looking for a pattern in their derivatives,
- find Taylor series for functions by differentiating, integrating, or substituting in a known Taylor series.

REVIEW

- Review sigma notation and the ratio and root tests.

READING

- Read §11.6 in the textbook, but skip the section on "Power series solutions of differential equations" starting halfway down page 574 to right before the section summary on page 576. Read the section summary.
- Read §11.7.

QUESTIONS

(1) Write the following sum in sigma notation: $-1 + 3 - 5 + 7 - 9 + \cdots$

SOLUTION:

$$\sum_{n=0}^{\infty} (-1)^n (2n+1)$$

(2) Suppose that $\sum a_n x^n$ converges for $x = 5$. Must it also converge for $x = 4$? For $x = -3$?

SOLUTION: The power series $\sum a_n x^n$ is centered at $x = 0$. Because the series converges for $x = 5$, the radius of convergence must be at least 5 and the series converges absolutely at least for the interval $|x| < 5$. Both $x = 4$ and $x = 3$ are inside this interval, so the series converges for $x = 4$ and for $x = ?3$.

(3) Write down Taylor series for the following functions:

(a) e^x

SOLUTION:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

(b) $\sin(x)$

SOLUTION:

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

(c) $\cos(x)$

SOLUTION:

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$