REVIEW SOLUTIONS

3 August 2018

NAME: SOLUTIONS

(1) Find the area enclosed by the curves $y = \sin(x)$ and $y = \cos(x)$ for $0 \le x \le \pi/2$.

ANSWER: $2\sqrt{2} - 2$.

(2) Does the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$ converge or diverge?

ANSWER: Converge.

(3) Compute $\displaystyle\int_0^1 \frac{2}{(x^2+1)(x+1)}\,dx$

ANSWER: $\dfrac{\ln(2)}{2} + \dfrac{\pi}{4}$

(4) Does the series $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{n^2+n}{n^2+2n+1}}$ converge or diverge?

ANSWER: Diverge.

(5) Compute $\displaystyle\int \frac{1}{x^2\sqrt{x^2+1}}\,dx$.

ANSWER: $-\dfrac{\sqrt{x^2+1}}{x} + C$

(6) Does the series $\displaystyle\sum_{n=1}^{\infty} \sin(1/n)$ converge or diverge?

ANSWER: Diverge. Use limit comparison test with the harmonic series.

(7) Find b such that the arc length of the curve $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = b$ has length $\frac{14}{3}$.

ANSWER: $b = 3$

(8) Find the interval of convergence of $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n}(x-1)^n$.

ANSWER: $\left[\frac{2}{3}, \frac{4}{3}\right)$

(9) Find the interval of convergence of $f'(x)$, where $f(x) = \sum_{n=1}^{\infty} \frac{3^n}{n}(x-1)^n$.

ANSWER: $\left(\frac{2}{3}, \frac{4}{3}\right)$

(10) Does the sequence $a_n = (-1)^n \frac{n}{n^2+1}$ converge or diverge?

ANSWER: Converges to zero.

(11) Does the sequence $a_n = \frac{n^2+2e^n}{n^3+e^n}$ converge or diverge?

ANSWER: Converges to 2.

(12) Compute $\int \cos(\sqrt{x})\, dx$

ANSWER: $2\sqrt{x}\sin(\sqrt{x}) + 2\cos(\sqrt{x}) + C$

(13) Does the series $\sum_{n=1}^{\infty} \frac{2^n}{(n!)^2}$ converge or diverge?

ANSWER: Converge.

(14) Find the Maclaurin series of $f(x) = \int_0^x \frac{1}{1+t^4}\, dt$.

ANSWER: $\sum_{n=0}^{\infty} \frac{(-1)^n}{4n+1} x^{4n+1}$