Math 4410 Discussion questions, Sept. 9, 2019

Throughout these problems G is a graph with a finite number of vertices and edges.

(1) Let G be a simple graph with at least two vertices. Prove that there are two distinct vertices x, y of G such that $\deg(x) = \deg(y)$.

(2) Let $n = |V(G)|$. Consider the following three possible properties of G.
- $|E(G)| = n - 1$.
- G is connected.
- G has no polygons.

Prove that any two of these properties implies the third.

(3) Let T be a tree with at least two vertices. Prove that T has at least two monovalent vertices.

(4) (a) Let P_1 and P_2 be polygons which are subgraphs of a graph G. Prove that if $e \in E(P_1) \cap E(P_2)$, then there exists a third polygon P_3 in G such that $e \notin P_3$ and $E(P_3) \subseteq E(P_1) \cup E(P_2)$.
(b) Let T be a tree with $V(T) = [n]$. Let e be an edge in K_n which is not in T. Show that $T \cup \{e\}$ has a *unique* polygon.