Math 4410 Discussion questions, Dec., 2019

1. Let c_n be the number of lattice paths in $\mathbb{R}^2$ from $(0,0)$ to (n,n) such that $x_i \geq y_i$ for all i. Prove that

$$c_{n+1} = \sum_{i=0}^{n} c_i c_{n-i}.$$

2. Prove that $c_n = \frac{1}{n+1}\binom{2n}{n}$.

3. Let a_n be a sequence so that there exists k and $q_1, \ldots, q_k$ such that

$$a_n = q_1 a_{n-1} + q_2 a_{n-2} + \cdots + q_k a_{n-k}.$$

   Prove that $F(x) = \sum_{n=0}^{\infty} a_n x^n$ equals $P(x)/Q(x)$ for some polynomials $P(x)$ and $Q(x)$.