Math 4410 Discussion questions, Nov. 1, 2019

(1) Explain why allowing loops and/or parallel edges does not change the problem of finding the maximum flow in directed graph.
(2) Use Theorem 7.1 to prove Theorem 5.1. Explain how our original proof of Theorem 5.1 is actually a particular case of the Ford-Fulkerson algorithm to find a maximum flow via special paths.
(3) Formulate and describe how to solve maximum flow problems for directed graphs with more than one source or sink.
(4) Let G be a directed graph without loops and let x, y be two distinct vertices of of G. A set of directed paths $\{P_1, \ldots, P_m\}$ in G from x to y are *edge disjoint* of no two different paths have an edge in common. Let $\kappa(G, x, y)$ be the maximum number of paths in a set of edge disjoint paths from x to y in G.

    Now let $C(G, x, y)$ be the minimum number of edges which must removed from G so that there are no directed paths in G from x to y. If there are no directed paths from x to y, then $C(G, x, y) = 0$. If you remove less than $\kappa(G, x, y)$ edges then one of the paths in a set of $\kappa(G, x, y)$ edge disjoint paths from x to y will remain intact. Hence, $\kappa(G, x, y) \leq C(G, x, y)$.

    Prove $C(G, x, y) \leq \kappa(G, x, y)$, and hence $C(G, x, y) = \kappa(G, x, y)$.