Math 4410 HW 11 - Will not be collected

1. Let C be a convex n-gon in the plane. How many ways are there to draw chords which divide the n-gon into triangles? For a square there are two - one for each of the two diagonals. For a pentagon there are five - for each vertex v of the pentagon there is one way to divide the pentagon into triangles by having two chords starting at v.

2. Let a_n be a sequence so that there exists k and $q_1, \ldots, q_k$ such that

$$a_n = q_1 a_{n-1} + q_2 a_{n-2} + \cdots + q_k a_{n-k}.$$

   Prove that $F(x) = \sum_{n=0}^{\infty} a_n x^n$ equals $P(x)/Q(x)$ for some polynomials $P(x)$ and $Q(x)$.

3. Let a_n be as defined in HW 10 problem # 3. Let $F(x) = \sum_{n=0}^{\infty} a_n \frac{x^n}{n!}$ be the associated exponential generating function.

   (a) Prove that $2F'(x) = (F(x))^2 + 1$.

   (b) Prove that $F(x) = \sec x + \tan x$.