

**Math 4410 HW 3 - Due Sept. 30, 2019 in class IN TEX**

1. Let  $G$  be a simple graph with subgraphs  $H_1, H_2$ . We write  $G = H_1 \cup H_2$  if  $V(G) = V(H_1) \cup V(H_2)$  and  $E(G) = E(H_1) \cup E(H_2)$ . Prove that  $\chi(G) \leq \chi(H_1) \cdot \chi(H_2)$ . (Hint: First show that you can assume that  $V(G) = V(H_1) = V(H_2)$ .)
2. Let  $G$  be a simple graph with  $V(G) = [n]$ . The complement of  $G$  is the simple graph whose vertex set is  $[n]$  and whose edges are the complement of the edges of  $G$ . Let  $\bar{G}$  be the complement of  $G$ . Prove that
$$\chi(G) + \chi(\bar{G}) \leq n + 1.$$
3. Let  $T$  and  $T'$  be trees with vertex set  $[n]$ . Prove that  $\chi_T(t) = \chi_{T'}(t)$  and provide an explicit formula for  $\chi_T(t)$ .