Math 4410 HW 5 - Due Oct. 25 in class

1. Let z be an integer (possibly negative). Define $z_\star = \max(1, z)$.

   (a) (warm up for the next question) Let $m_0 \le m_1 \le m_2$ be positive integers. Prove that there exists a bipartite graph G such that

   - $V(G)$ is the disjoint union of X and Y, and every edge has one end point in X and the other in Y.
   - $X = \{x_0, x_1, x_2\}$ and for all $0 \le i \le 2$, $\deg(x_i) = m_i$.
   - G has exactly

     $$m_0 \cdot (m_1 - 1)_\star \cdot (m_2 - 2)_\star$$

     complete matchings.

   (b) Now let $m_0 \le m_1 \le m_2 \le \cdots \le m_{n-1}$ be a sequence of positive integers. Prove there exists a bipartite graph G such that

   - $V(G)$ is the disjoint union of X and Y, and every edge has one end point in X and the other in Y.
   - $X = \{x_0, x_1, \ldots, x_{n-1}\}$ and for all $0 \le i \le n-1$, $\deg(x_i) = m_i$.
   - G has exactly

     $$\prod_{i=0}^{n-1} (m_i - i)_\star$$

     complete matchings.

2. Problem 5D of the text.

3. Problem 6B of the text.