Application: Production Costs.

A company manufactures three products. Its production expenses are divided into three categories. In each category, an estimate is given for the cost of producing a single item of each product. An estimate is also made of the amount of each product to be produced per quarter. These estimates are given in Tables 1 and 2:

	Product		
Expenses	**A**	**B**	**C**
Raw materials	0.10	0.30	0.15
Labor	0.30	0.40	0.25
Overhead	0.10	0.20	0.15

TABLE 1. Production Costs Per Item (dollars)

	Season			
Product	**Summer**	**Fall**	**Winter**	**Spring**
A	4000	4500	4500	4000
B	2000	2600	2400	2200
C	5800	6200	6000	6000

TABLE 2. Amount Produced Per Quarter

The company would like to present at their stockholders' meeting a single table showing the total costs for each quarter in each of the three categories: raw materials, labor, and overhead.

Solution. Consider the tables as represented by the matrices

$$M = \begin{bmatrix} 0.10 & 0.30 & 0.15 \\ 0.30 & 0.40 & 0.25 \\ 0.10 & 0.20 & 0.15 \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} 4000 & 4500 & 4500 & 4000 \\ 2000 & 2600 & 2400 & 2200 \\ 5800 & 6200 & 6000 & 6000 \end{bmatrix}$$

If we form the product MP, the first column will represent the costs for the summer quarter:

$$\begin{aligned} \text{Raw materials:} \quad & (0.10)(4000) + (0.30)(2000) + (0.15)(5800) = 1870 \\ \text{Labor:} \quad & (0.30)(4000) + (0.40)(2000) + (0.25)(5800) = 3450 \\ \text{Overhead:} \quad & (0.10)(4000) + (0.20)(2000) + (0.15)(5800) = 1670 \end{aligned}$$

The second column will represent the costs for the fall quarter:

$$\text{Raw materials:} \quad (0.10)(4500) + (0.30)(2600) + (0.15)(6200) = 2160$$
$$\text{Labor:} \quad (0.30)(4500) + (0.40)(2600) + (0.25)(6200) = 3940$$
$$\text{Overhead:} \quad (0.10)(4500) + (0.20)(2600) + (0.15)(6200) = 1900$$

Columns 3 and 4 of MP represent the costs for the winter and spring quarters.

$$MP = \begin{bmatrix} 1870 & 2160 & 2070 & 1960 \\ 3450 & 3940 & 3810 & 3580 \\ 1670 & 1900 & 1830 & 1740 \end{bmatrix}$$

The entries in the first row of MP represent the total cost of raw materials for each of the four quarters. The entries in rows 2 and 3 represent the total cost for labor and overhead, respectively, for each of the four quarters. The yearly expenses in each of the columns may be added to obtain the total production costs for each quarter. Table 3 summarizes the total production costs.

	Season				
	Summer	**Fall**	**Winter**	**Spring**	**Year**
Raw materials	1870	2160	2070	1960	8060
Labor	3450	3940	3810	3580	14780
Overhead	1670	1900	1830	1740	7140
Total costs	6990	8000	7710	7280	29980

TABLE 3. Total Production Costs (dollars)

Problem 1. A toy manufacturer makes toy airplanes, boats, and cars. Each toy is fabricated in a factory F_1 in Taiwan and then assembled in factory F_2 in the U.S. The total cost of each product consists of the manufacturing cost and the shipping cost. Then the costs at each factory (in \$US) can be described as

$$F_1 = \begin{bmatrix} 0.32 & 0.40 \\ 0.50 & 0.80 \\ 0.70 & 0.20 \end{bmatrix} \begin{array}{l} \text{Airplanes} \\ \text{Boats} \\ \text{Cars} \end{array}$$

with columns labeled Manufacturing Costs and Shipping Costs,

and

$$F_2 = \begin{bmatrix} 0.40 & 0.60 \\ 0.50 & 0.50 \\ 1.30 & 0.20 \end{bmatrix} \begin{array}{l} \text{Airplanes} \\ \text{Boats} \\ \text{Cars} \end{array}$$

with columns labeled Manufacturing Costs and Shipping Costs.

Find a matrix that gives the total manufacturing and shipping costs for each product.

Warning: you need to use some practical reasoning here;
do not mimic the previous example exactly!