Fractal spectrum and quasi-periodic potential

ERIC AKKERMANS
PHYSICS-TECHNION

Benefitted from discussions and collaborations with:

Evgeni Gurevich, Technion
Jacqueline Bloch, Marcoussis
Dimitri Tannese, Marcoussis
Julien Gabelli Orsay
Gerald Dunne UConn.
Sasha Teplyaev, UConn.

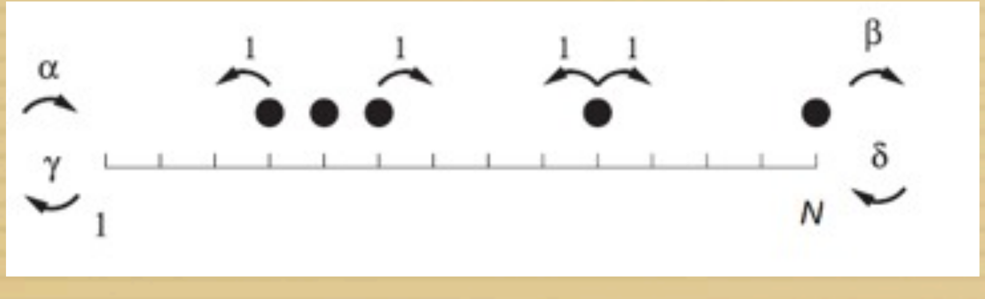
5th Cornell Conference on Fractals,
June 2014

Large deviations and some stochastic processes on graphs and fractals

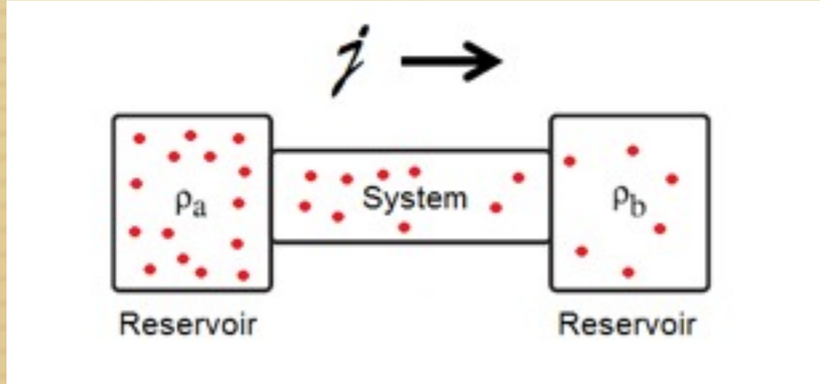
Elaborations

Large deviations and some stochastic processes on graphs and fractals

The Simple Symmetric Exclusion Process (SSEP)



$\Leftrightarrow$



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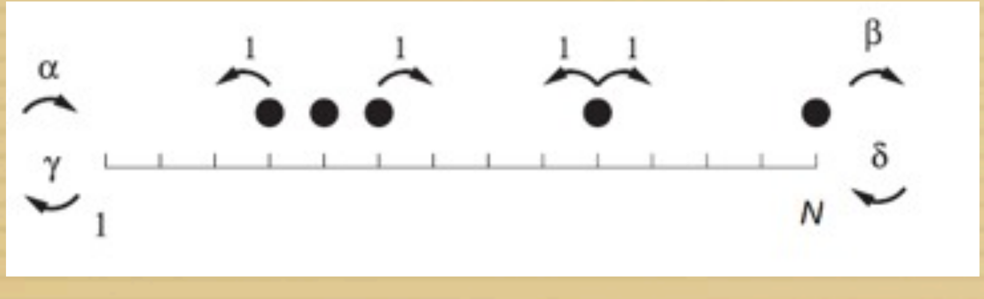

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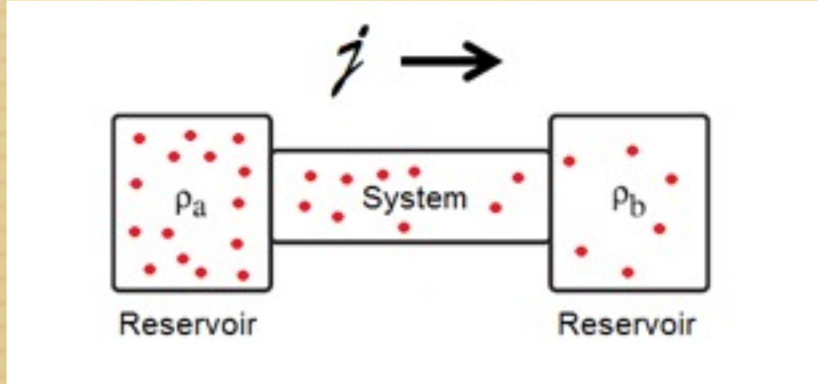

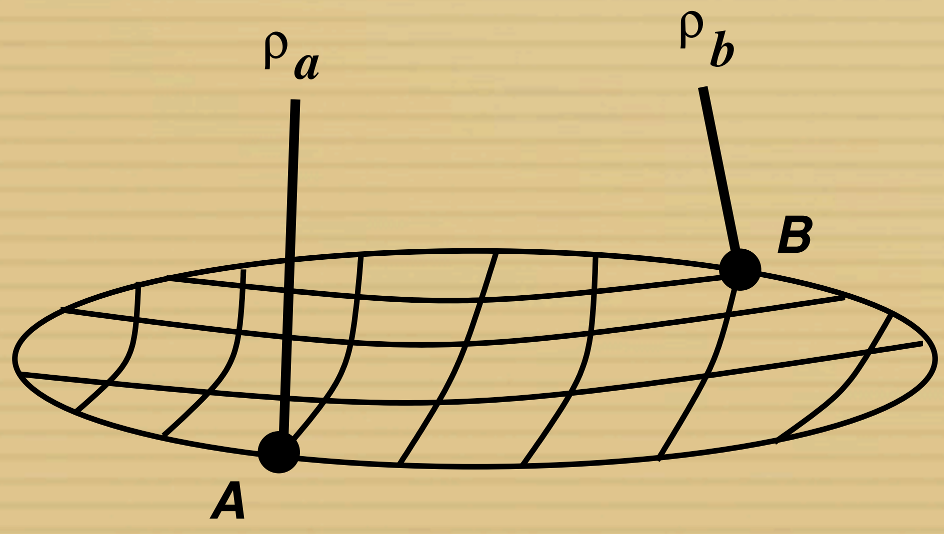

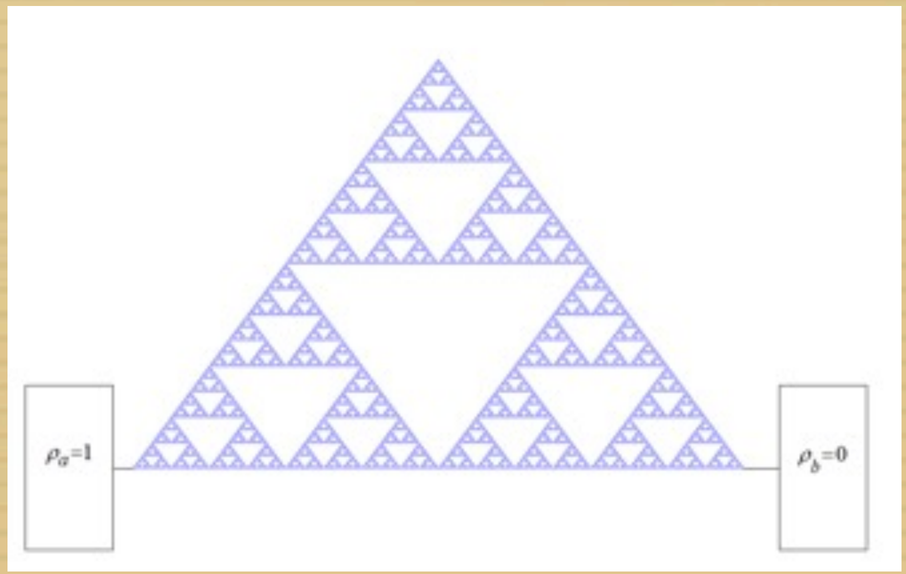

Back to quantum physics on fractals

A large variety of problems are conveniently described in terms of spectral classes

(absolutely continuous / singular-continuous / point spectrum):

- Anderson localization
- Quantum and classical wave diffusion
- Random magnetism
- …

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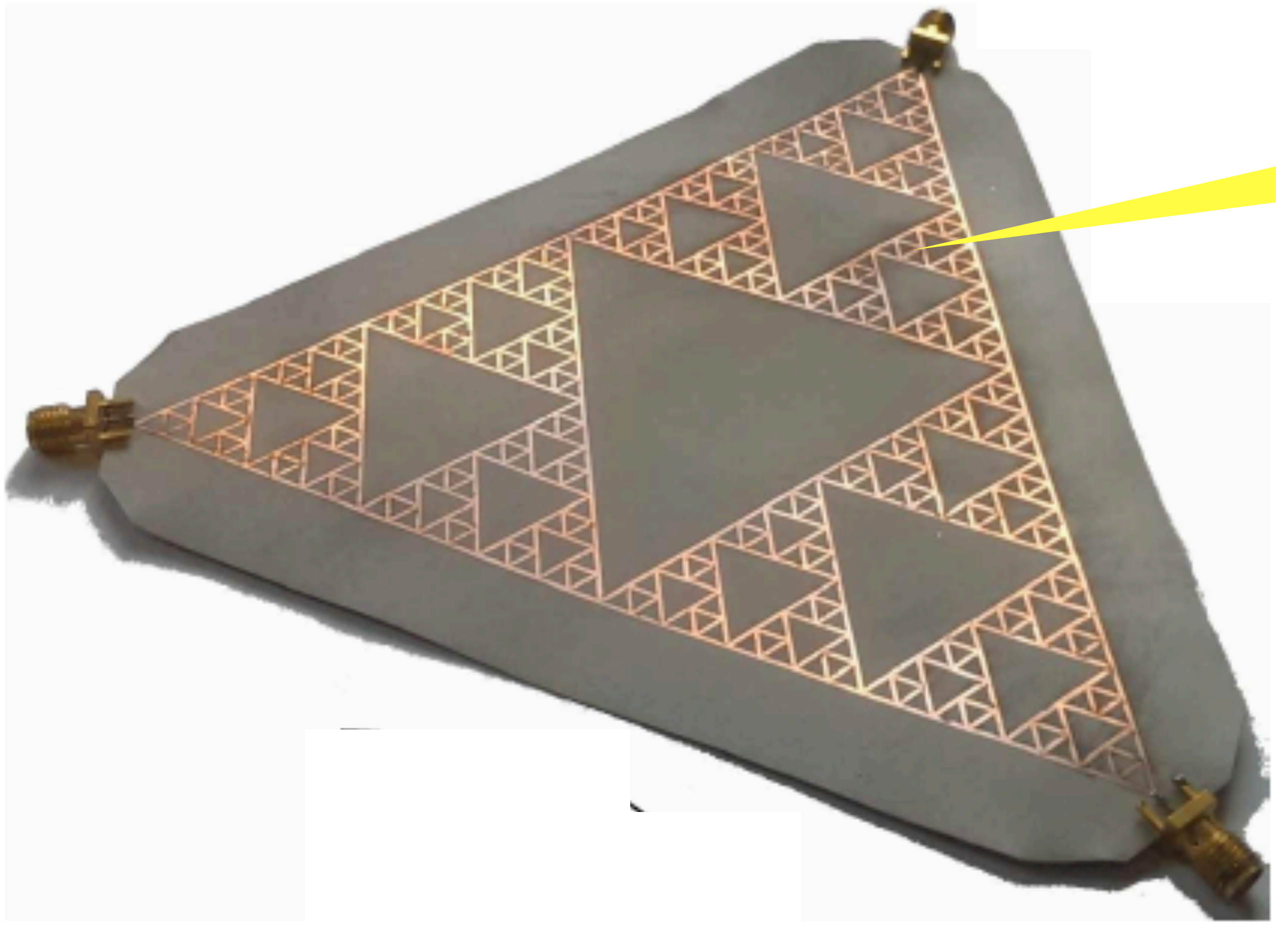
absolutely continuous

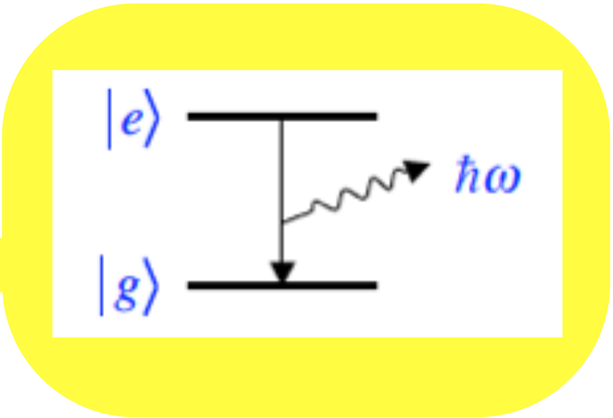
What about a fractal spectrum ?

pectrum

An interesting problem in that context

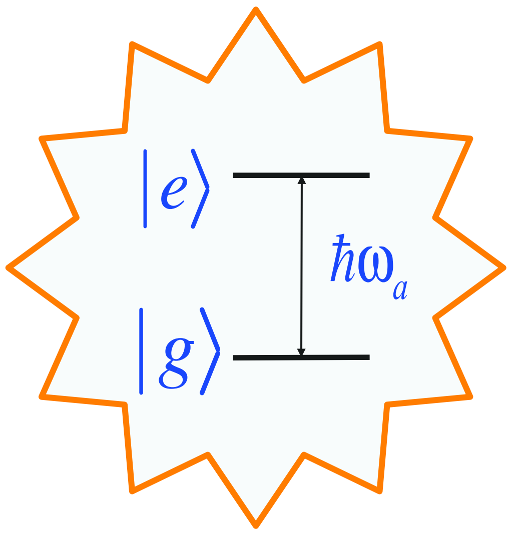
Spontaneous emission from a fractal QED cavity/spectrum





E.A. G. Dunne, A. Teplyaev, EPL, 2
E.A. and G. Dunne, PRL 2010
E.A and E. Gurevich, EPL,2013

Spontaneous emission for different QED vacua



atom

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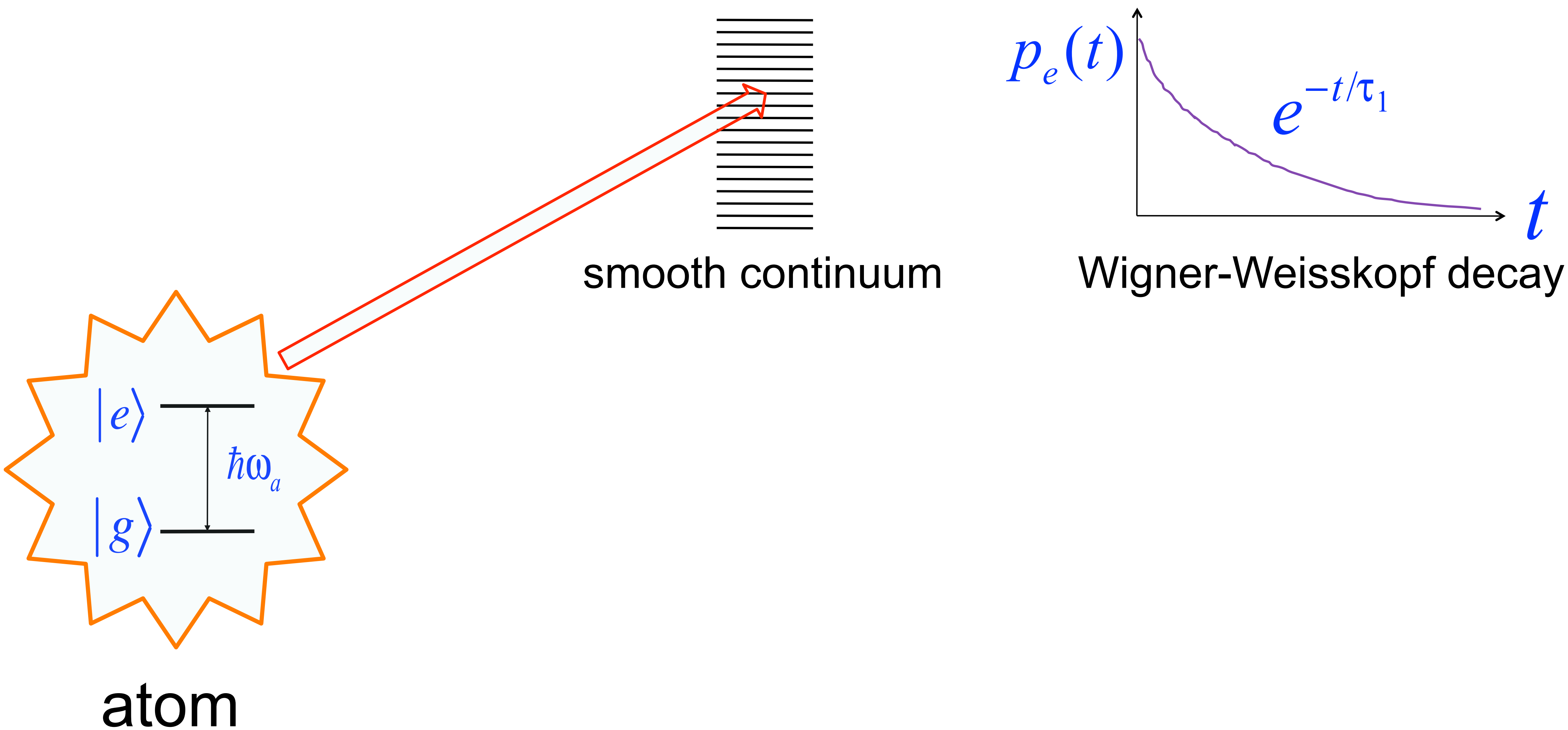

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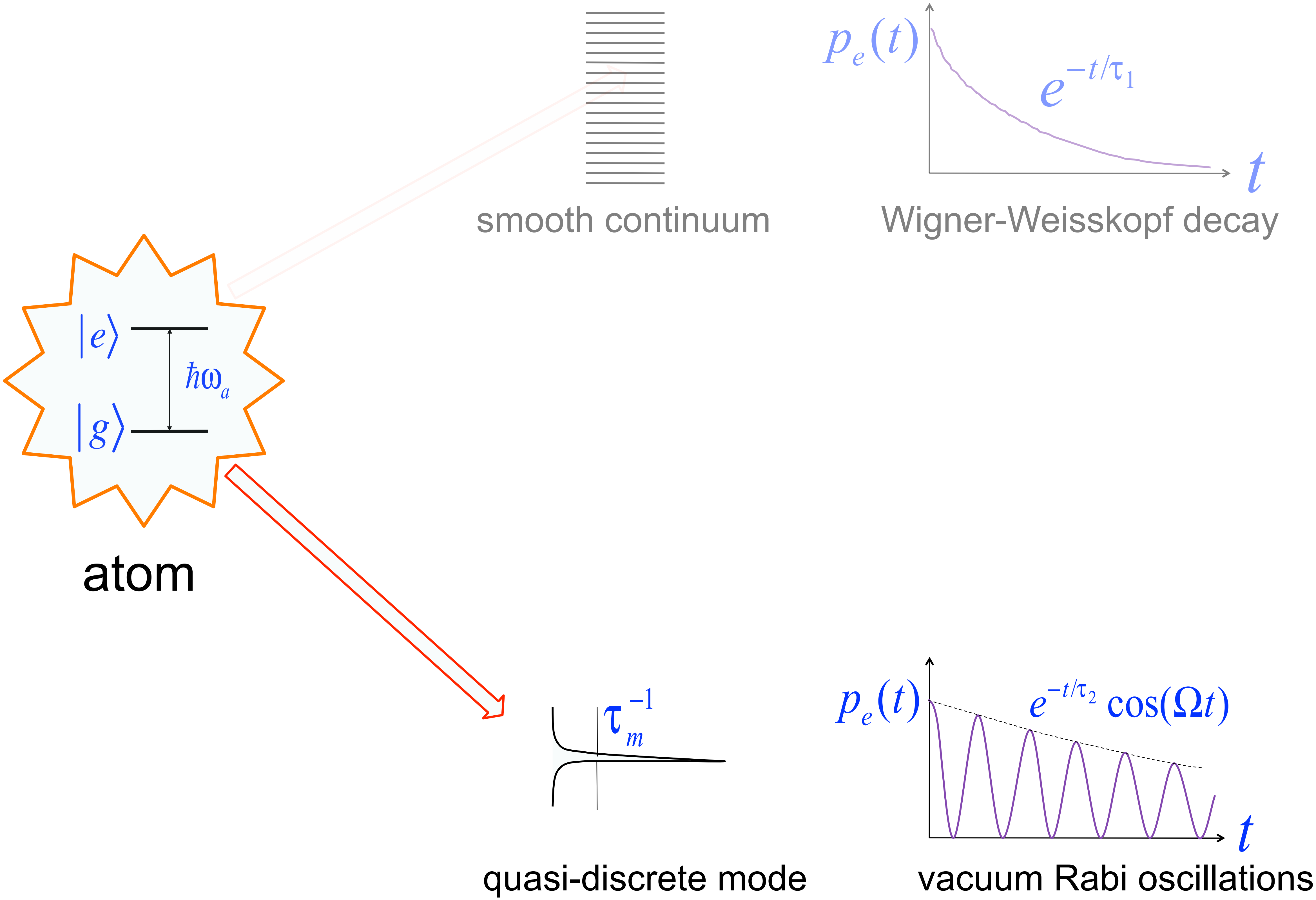

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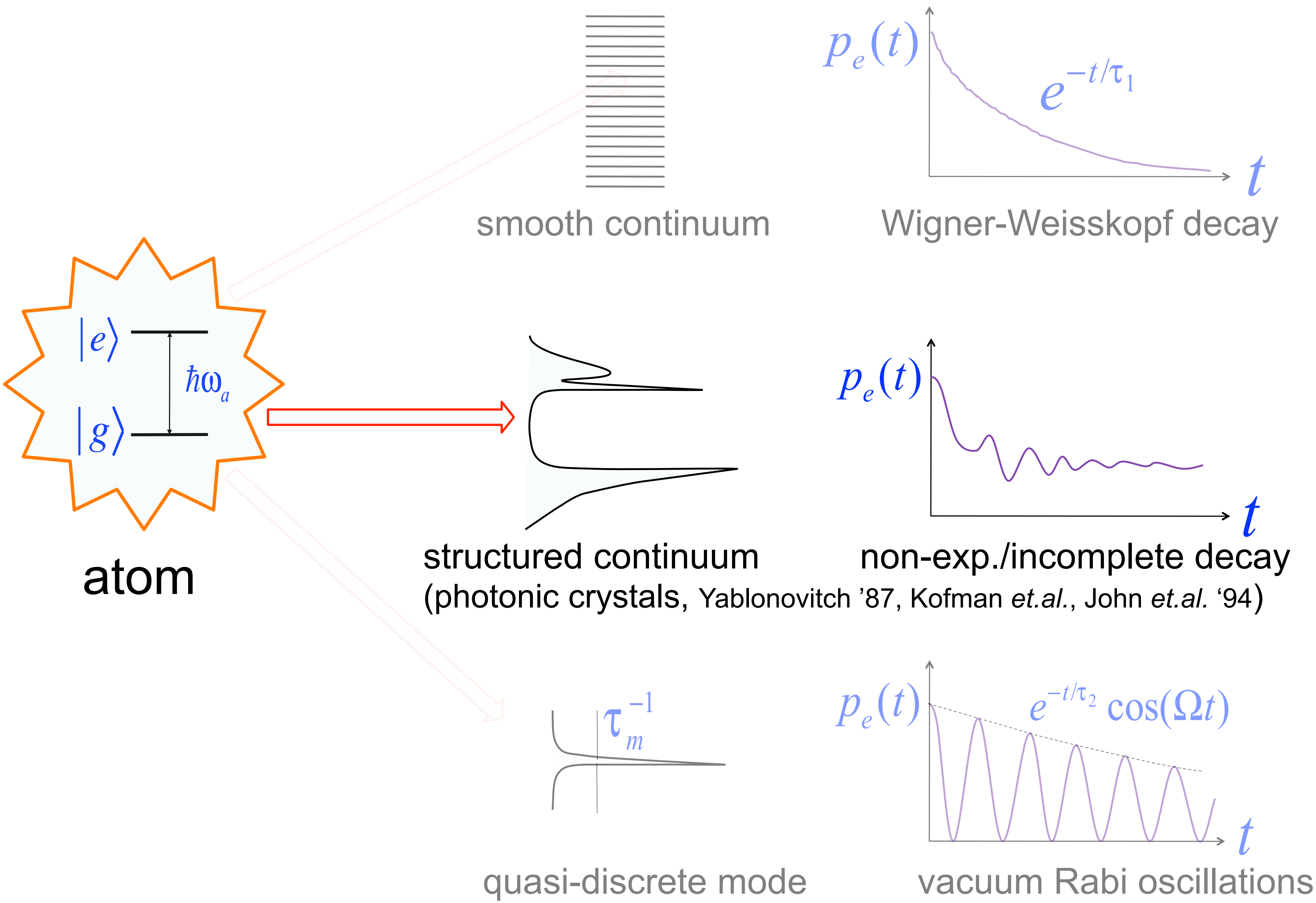

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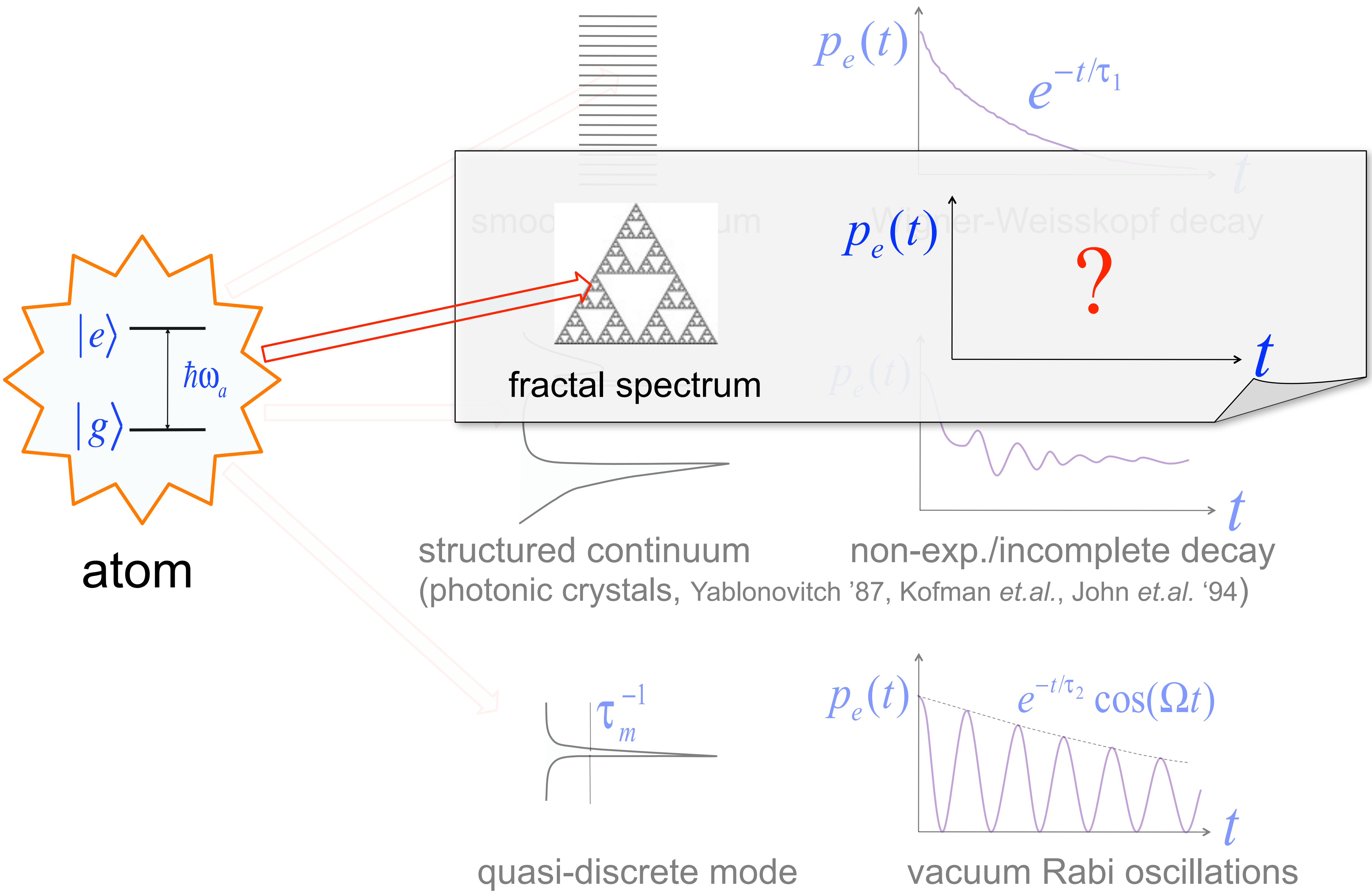
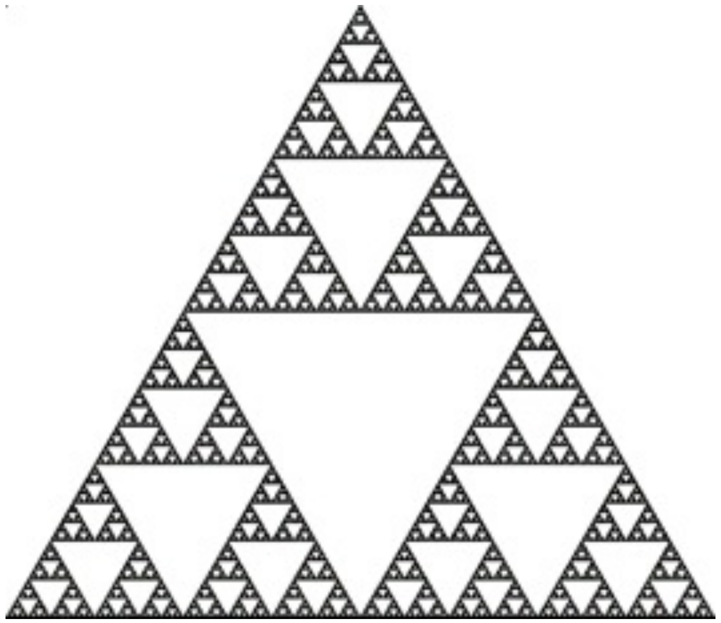


Fractal spectrum ?

Fractal $\leftrightarrow$ Self-similar



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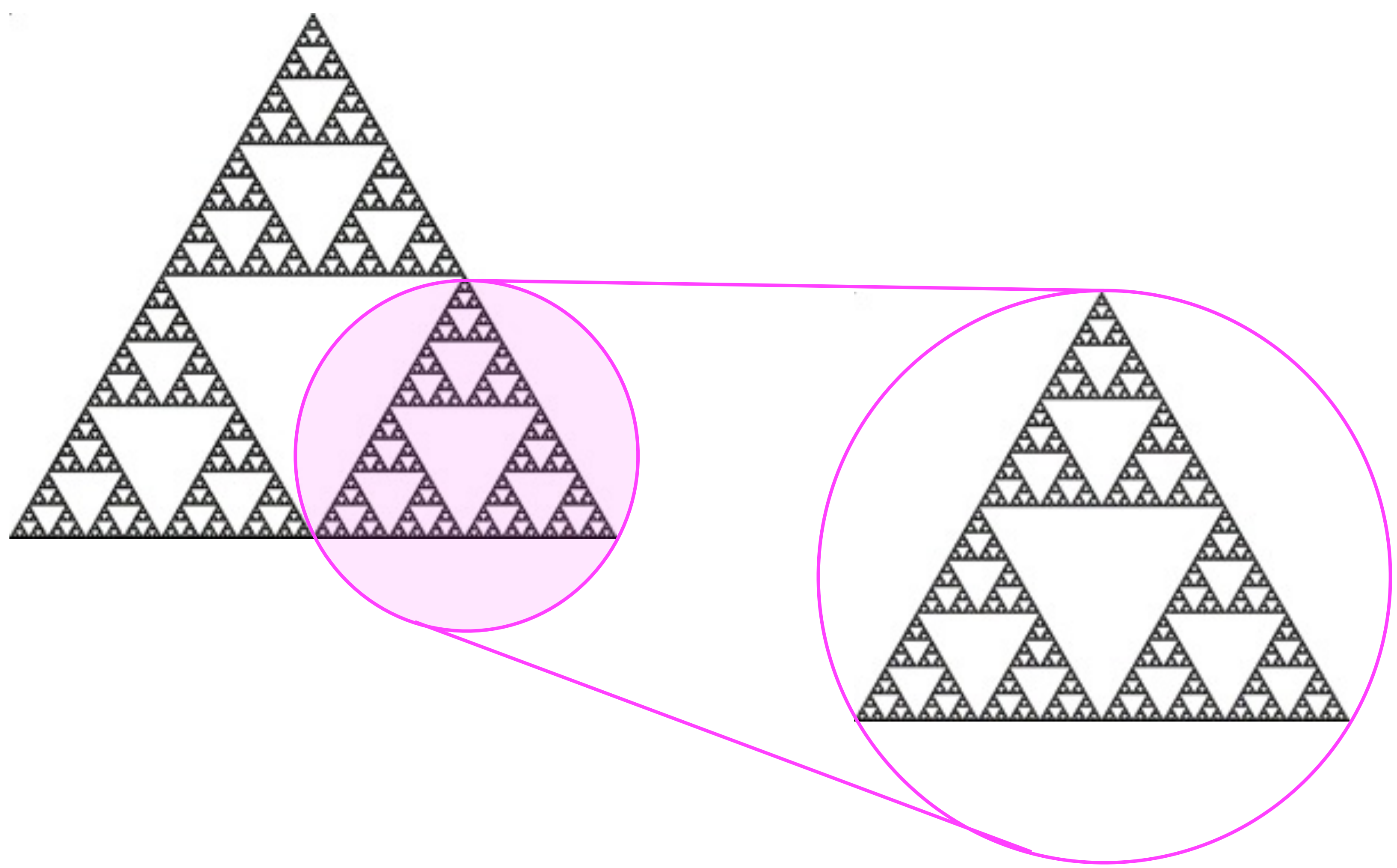

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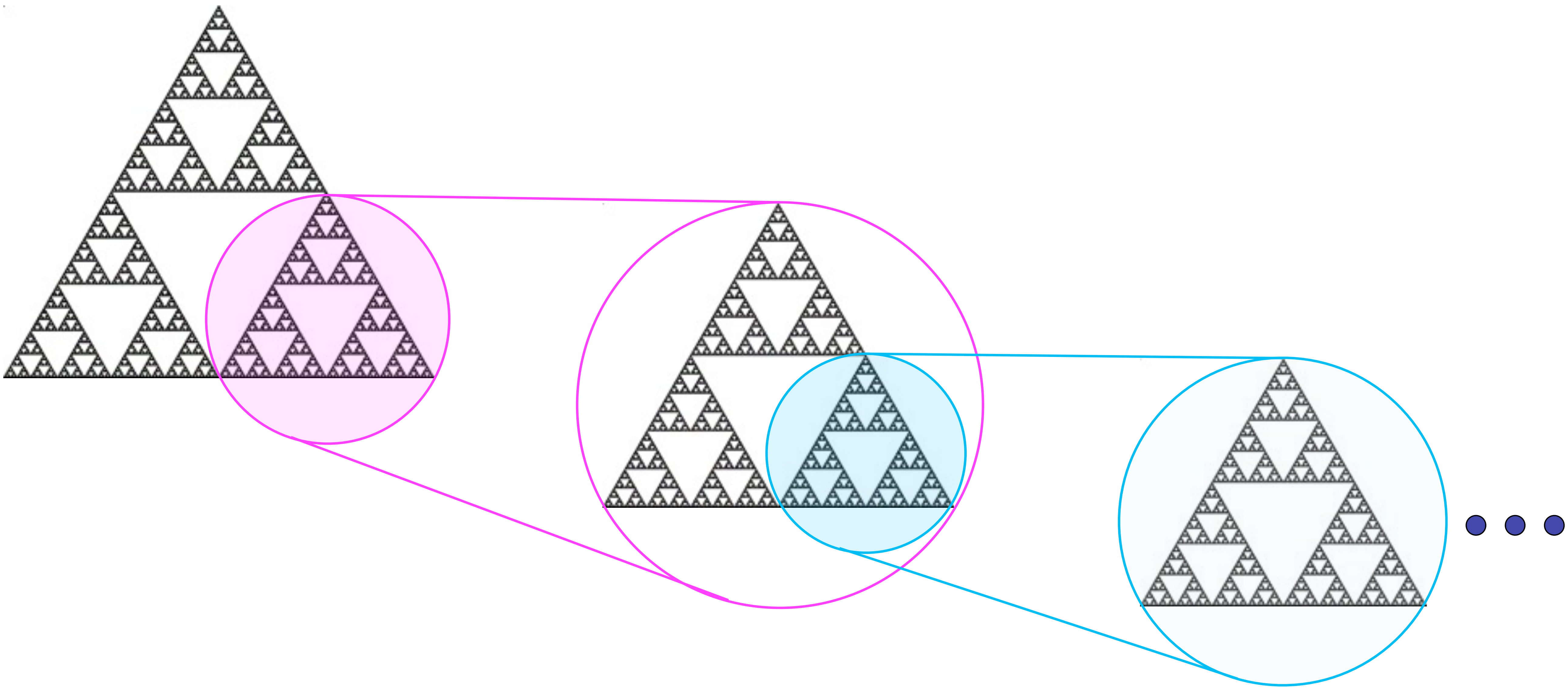
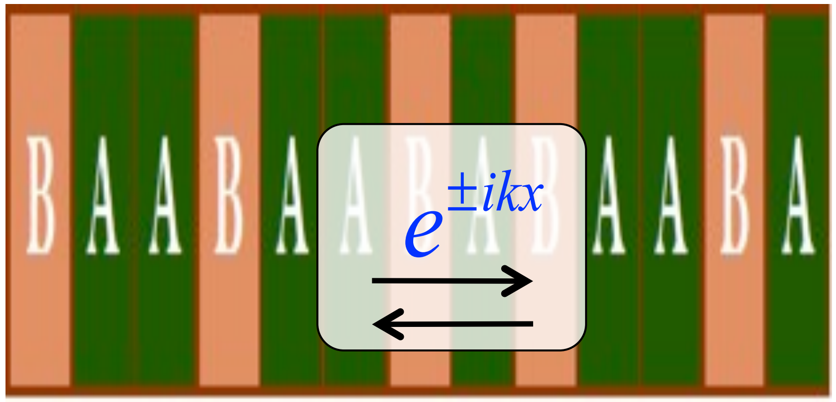


Discrete scaling symmetry

Fractal spectrum - an example

A quasi-periodic stack of dielectric layers of two types (n_A,n_B)



(Kohmoto *et. al.*, '87)

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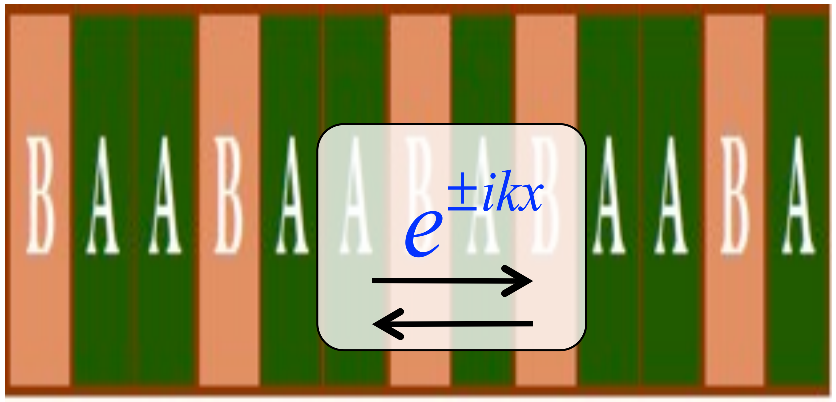
Fibonacci sequence: $S_{j\geq 2} = \left[S_{j-1} S_{j-2} \right], \quad S_0 = B, \quad S_1 = A$

A→AB→ABA→ABAAB→ABAABABA→…



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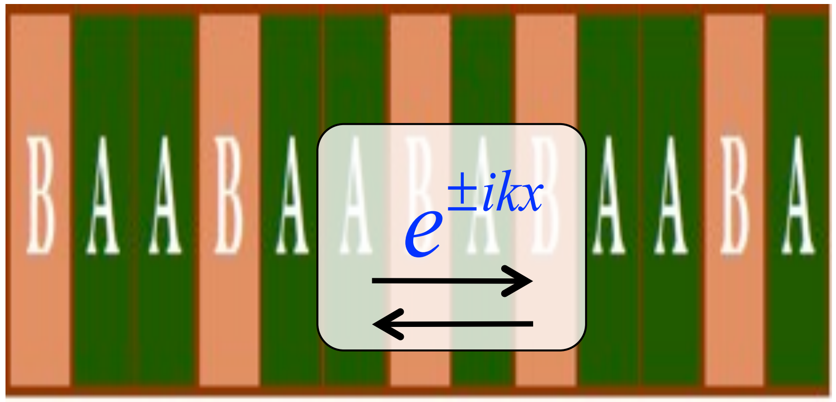

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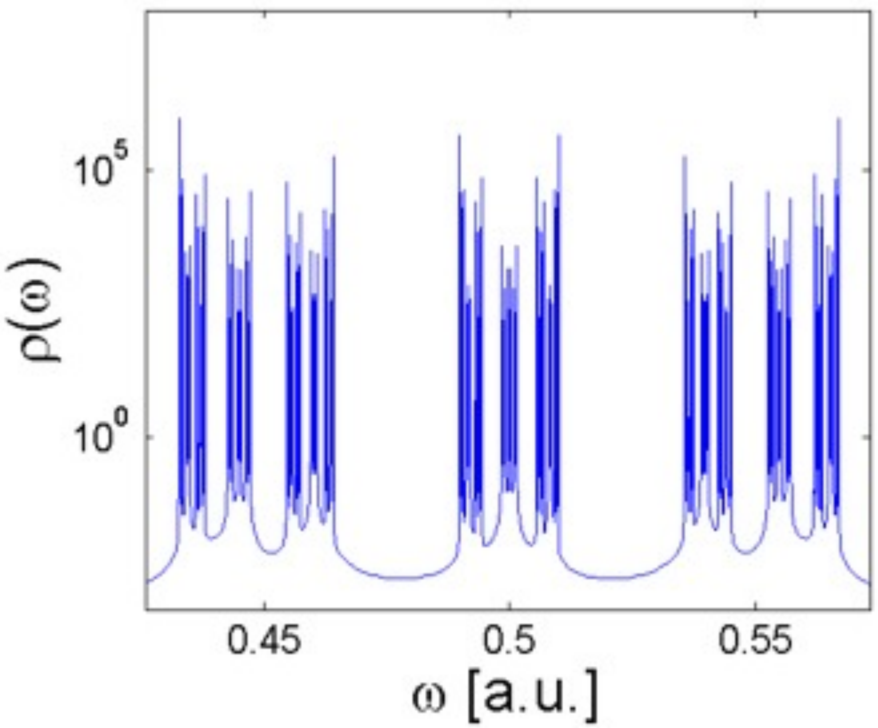
The density of modes $\rho(\omega)$:



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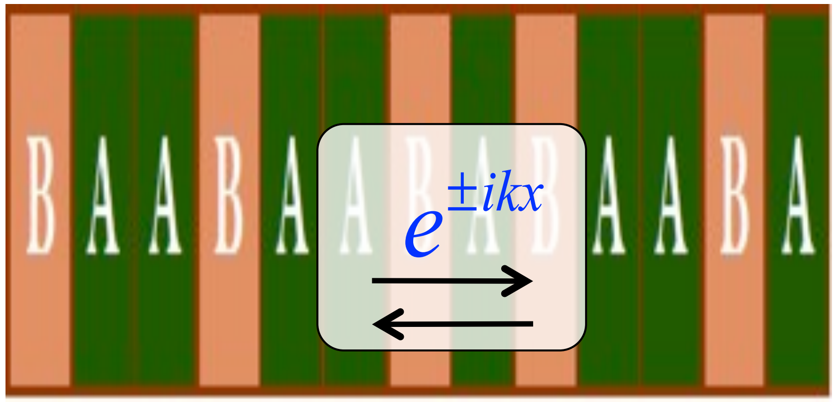

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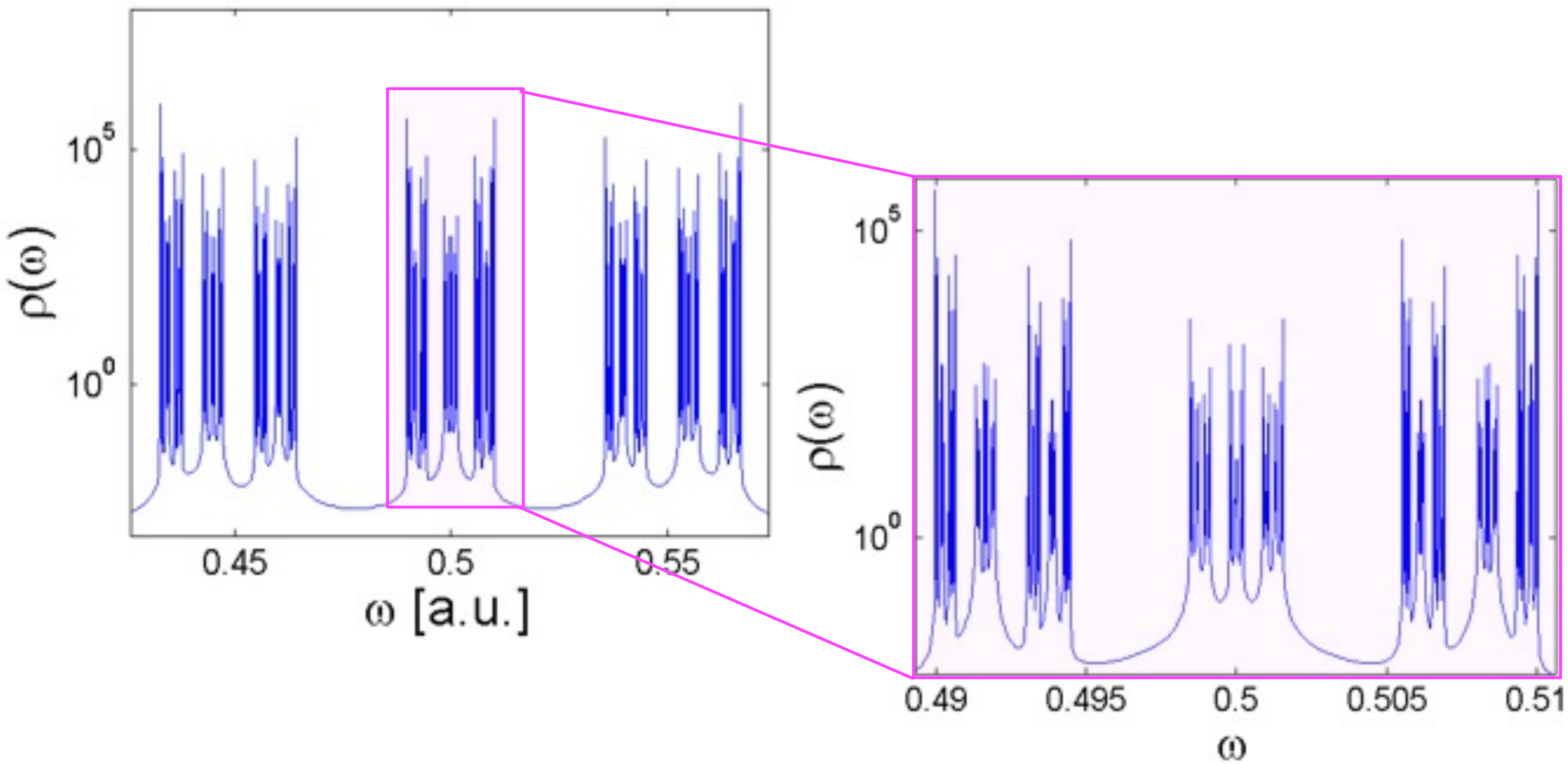

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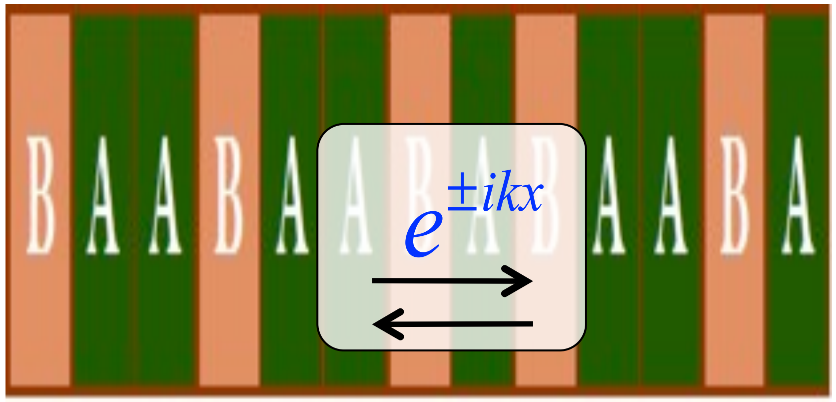

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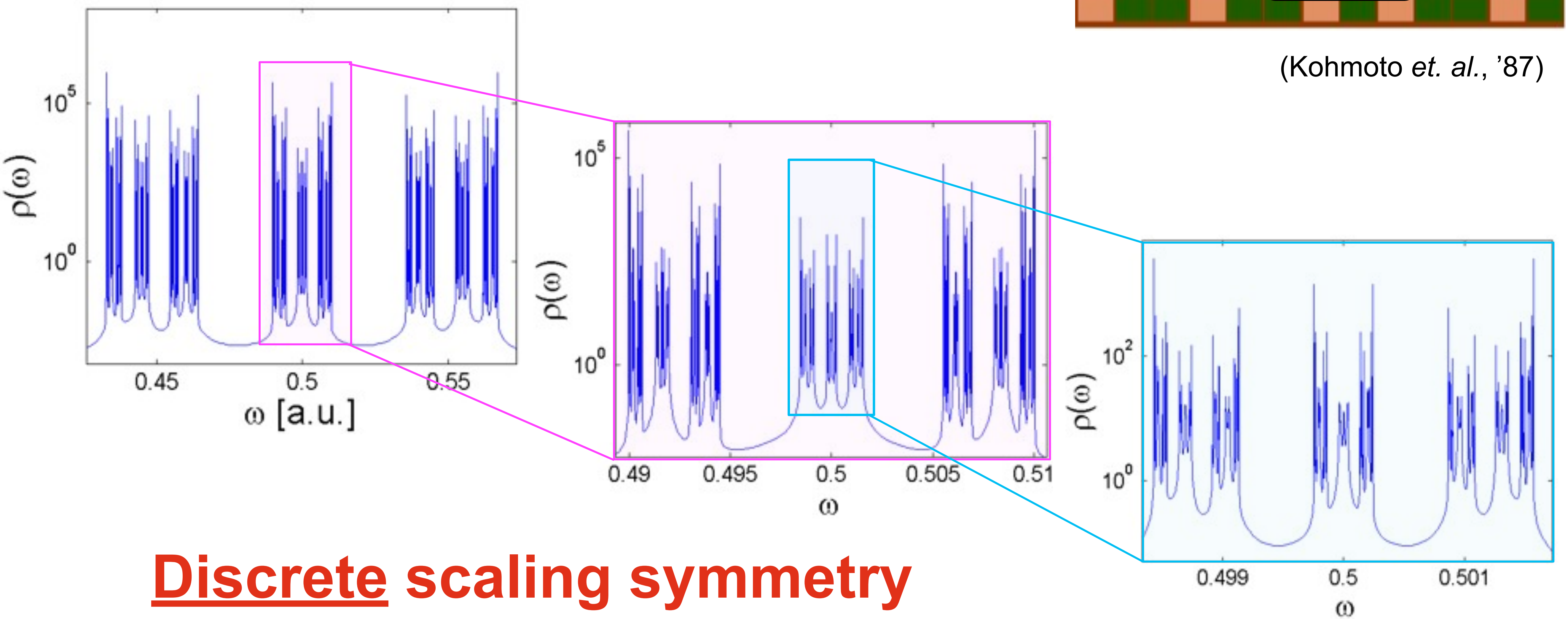
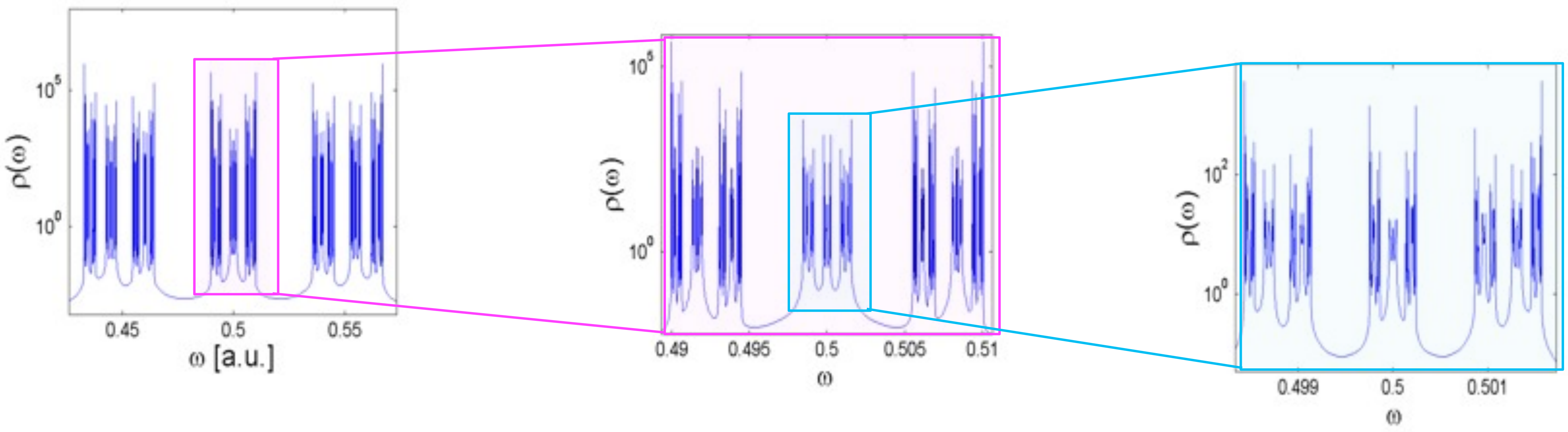


<u>Discrete</u> scaling symmetry

Discrete scaling symmetry: formal description



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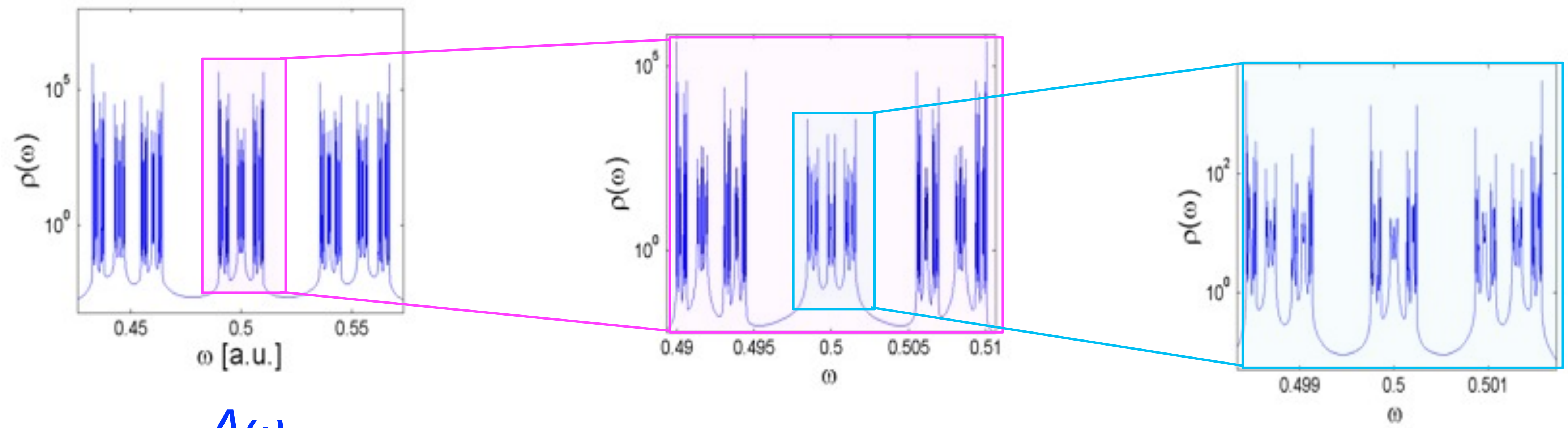

$\longrightarrow \Delta\omega \longleftarrow$

Counting function: $N_\omega(\Delta\omega) \equiv \int_{\omega}^{\omega+\Delta\omega} \rho(\omega')d\omega'$ = (# of states in $[\omega, \omega+\Delta\omega]$)

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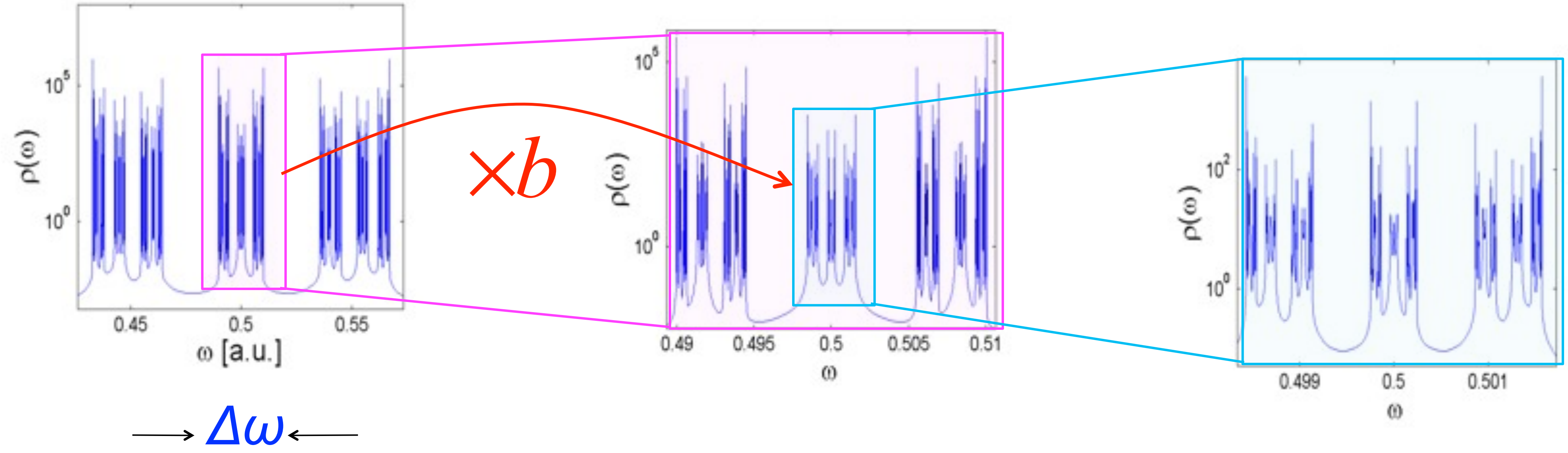

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$$N_\omega(b\ \Delta\omega) = a\ N_\omega(\Delta\omega)$$

b, a - fixed scaling factors

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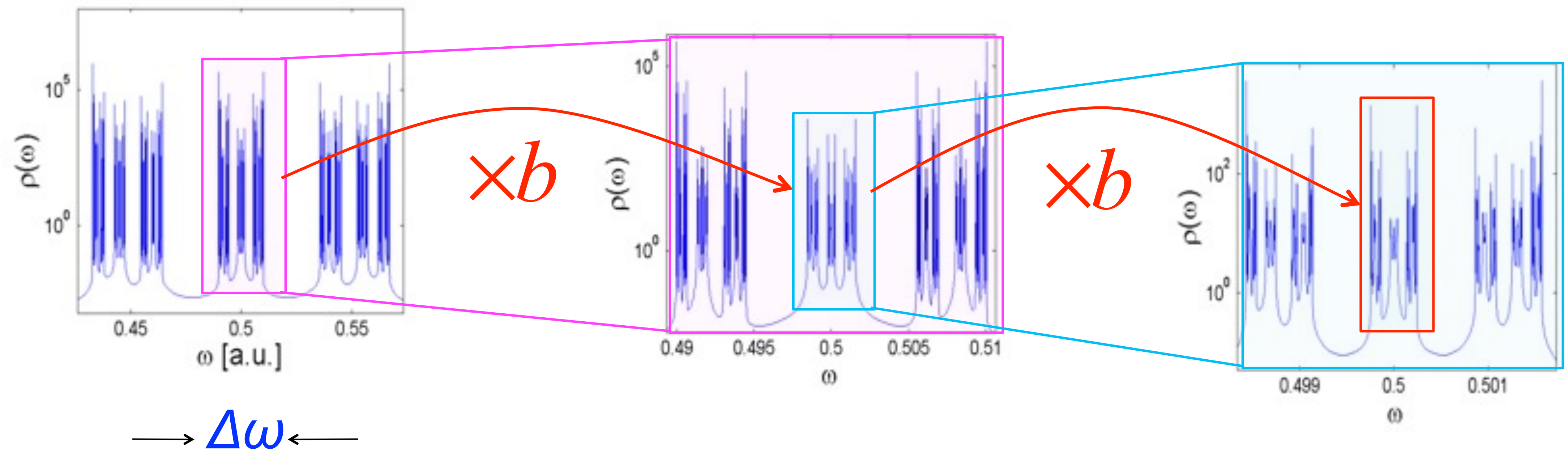

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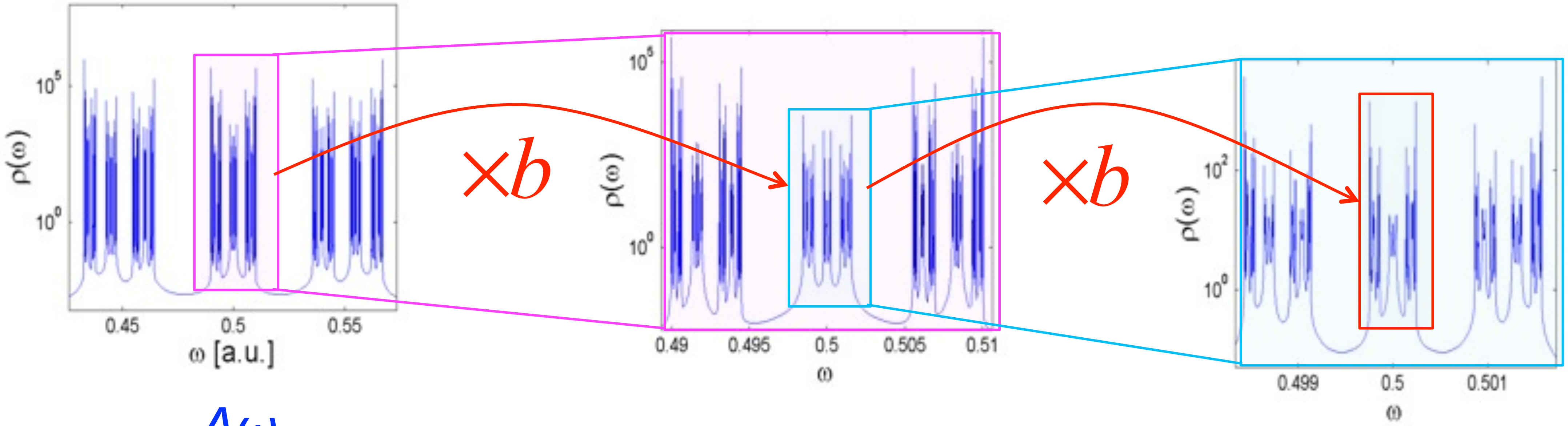

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b, a - fixed scaling factors

$\Rightarrow$ **Discrete scaling symmetry**

Testing the discrete scaling symmetry

Scaling equation

$$N_\omega(b^p \Delta\omega) = a^p N_\omega(\Delta\omega), \qquad N_\omega(\Delta\omega) \equiv \int_{\omega}^{\omega+\Delta\omega} \rho(\omega')\, d\omega'$$

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has the following general solution (dimensionless ω):

$$N_\omega(\Delta\omega) = (\Delta\omega)^\alpha \times (\cdots) \quad , \quad \alpha = \frac{\ln a}{\ln b}$$

$0 \le \alpha \le 1$ - fractal exponent (absolutely continuous : $\alpha = 1$, pure-point : $\alpha = 0$)

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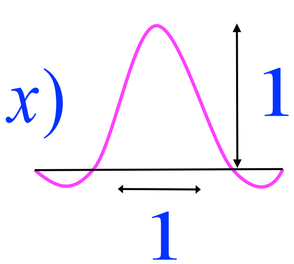
Similarly for the convolution of $\rho(\omega)$ with a window function $g(x)$



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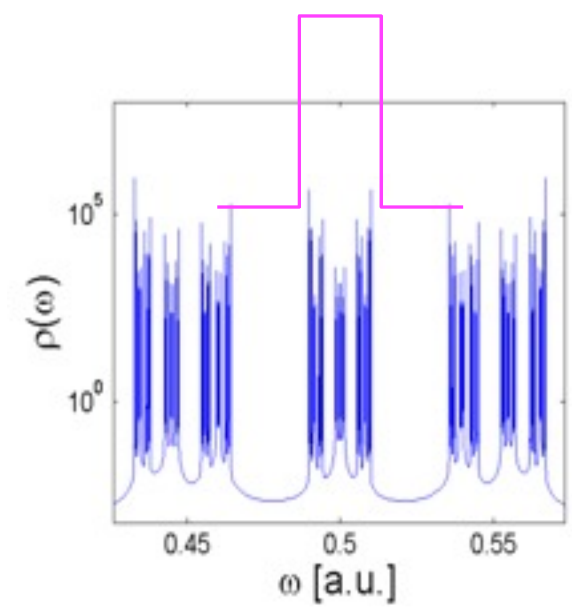

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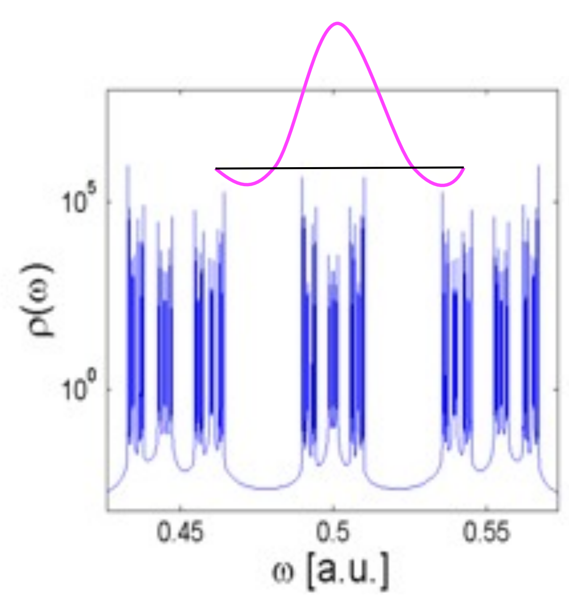

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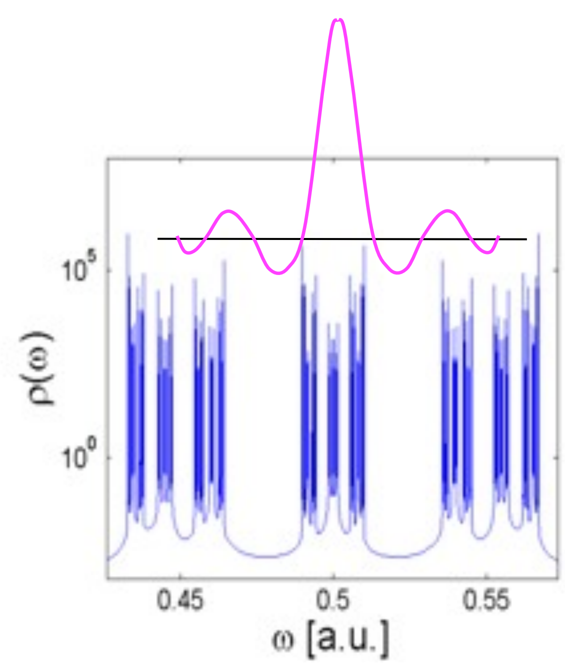
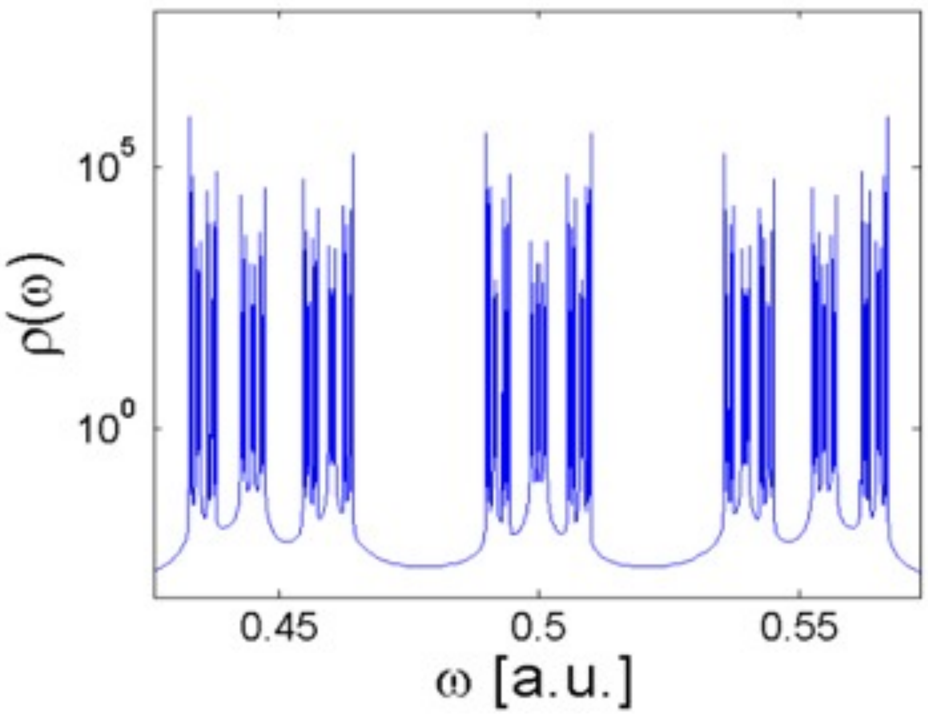


(Ghez and Vaienti, ’89: the wavelet transform of fractal measures)

Testing the discrete scaling symmetry - an example

A quasi-periodic dielectric stack



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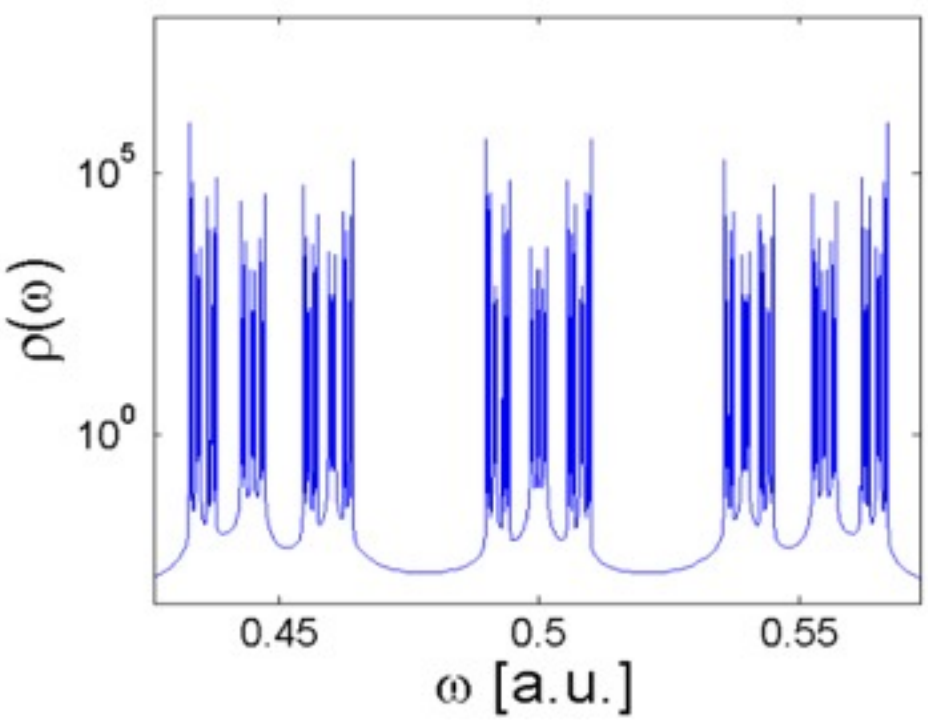

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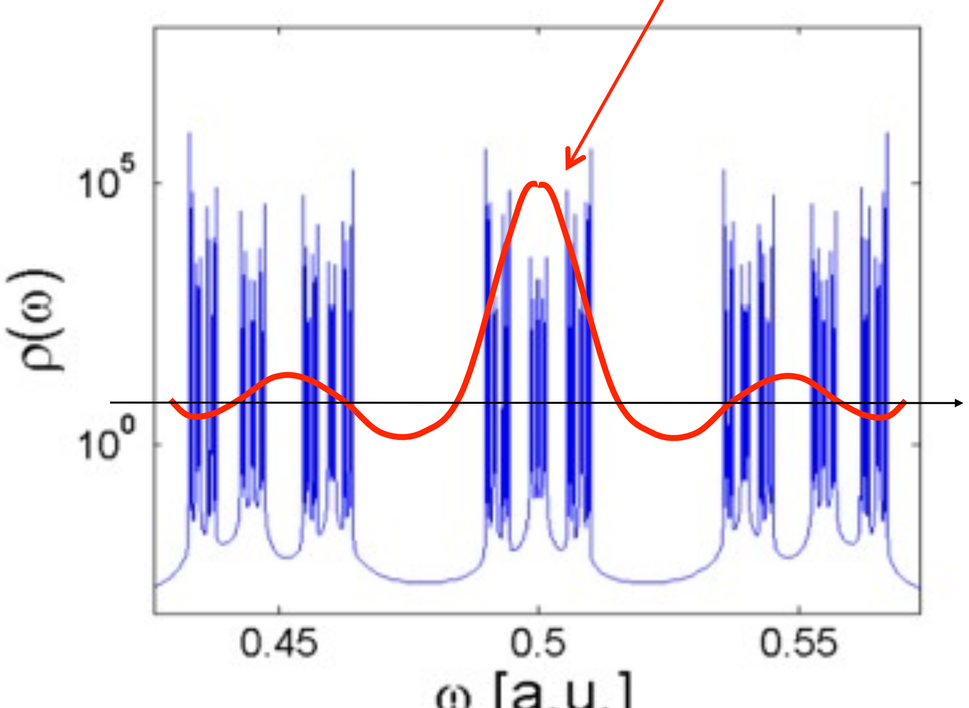

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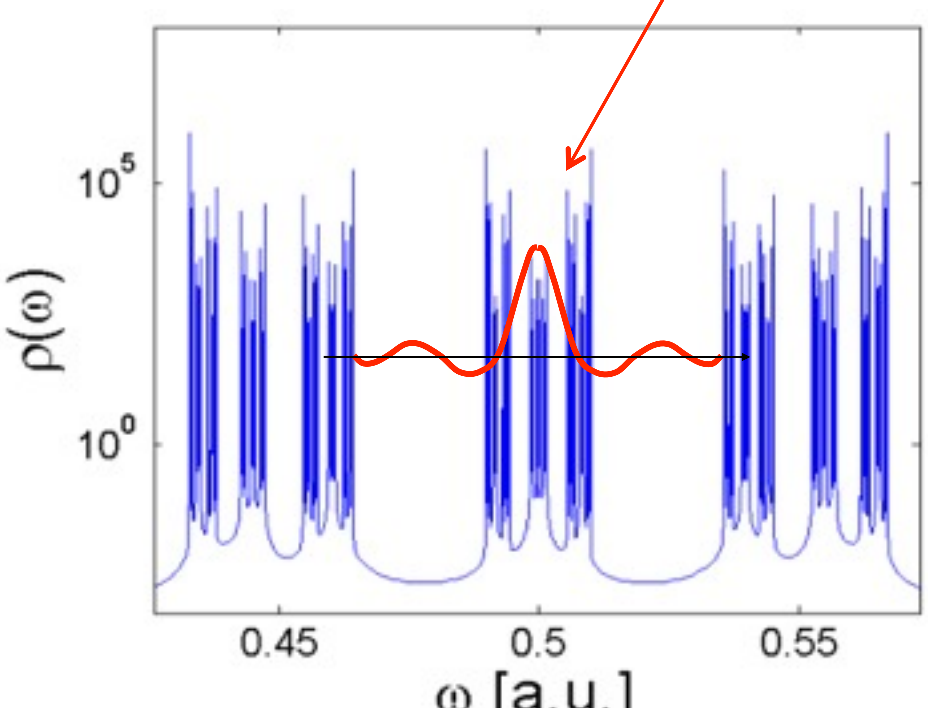

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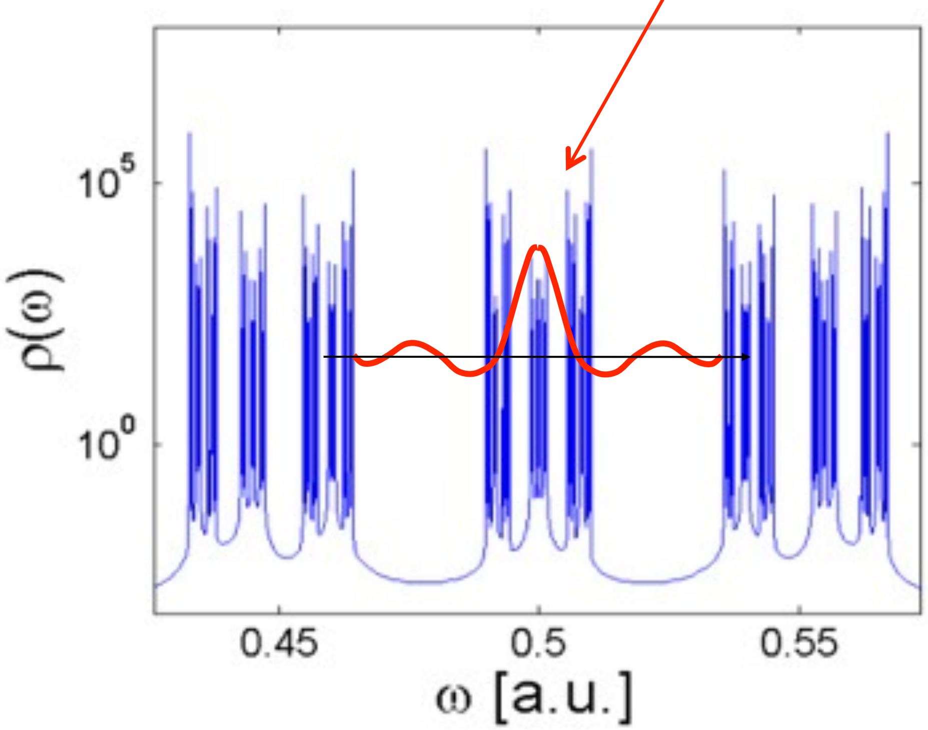
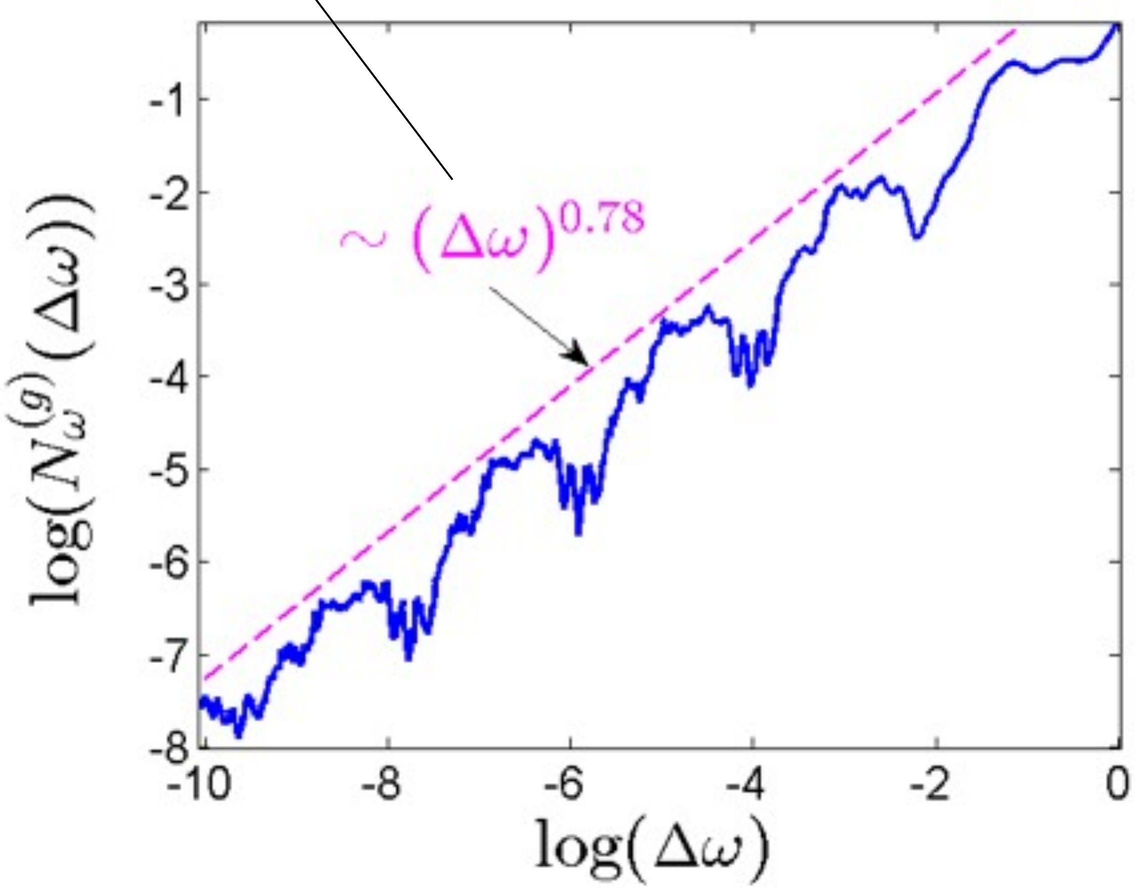


numerics



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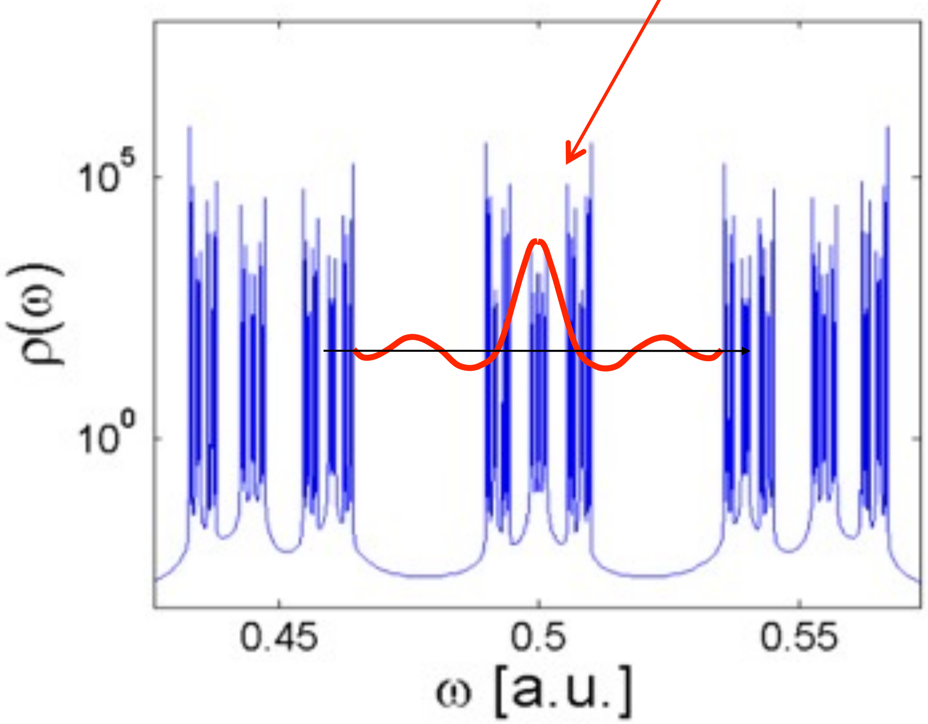

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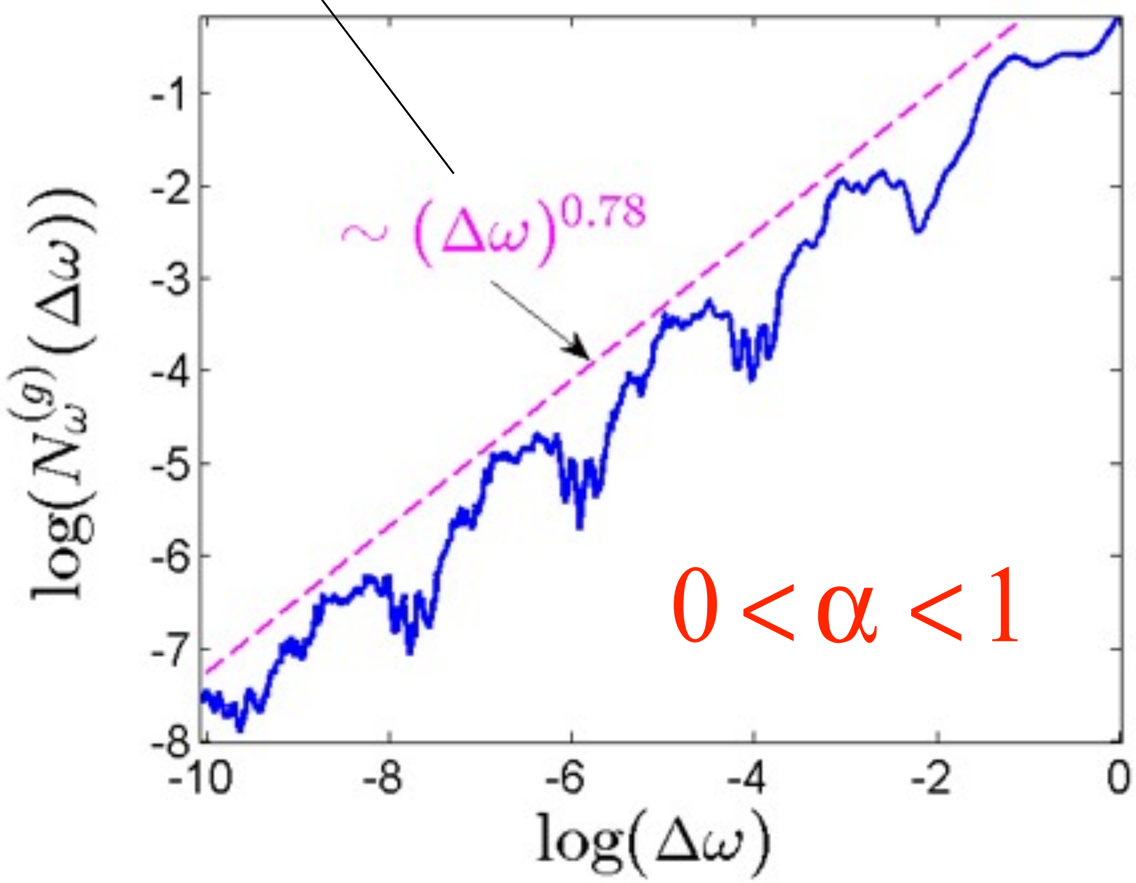

$\alpha = \alpha(n_A, n_B) = 0.777$ (Kohmoto *et. al.*, '87)

Experimental study of a fractal energy spectrum :

Cavity polaritons in a Fibonacci quasi-periodic potential

D. Tanese, J. Bloch, E. Gurevich, E.A. PRL, 2014.

The Fibonacci problem has a long and rich (theoretical and experimental) history.

(Kohmoto,Luck, Gellerman, Damanik, Bellissard,Simon,...)

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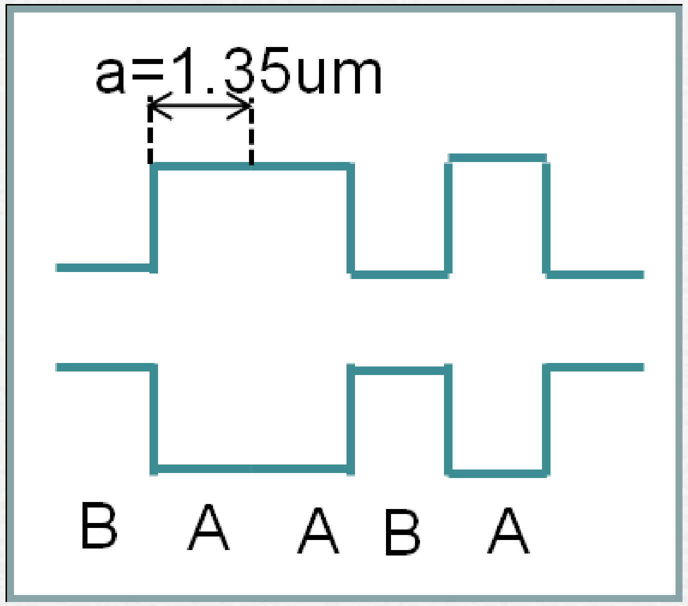
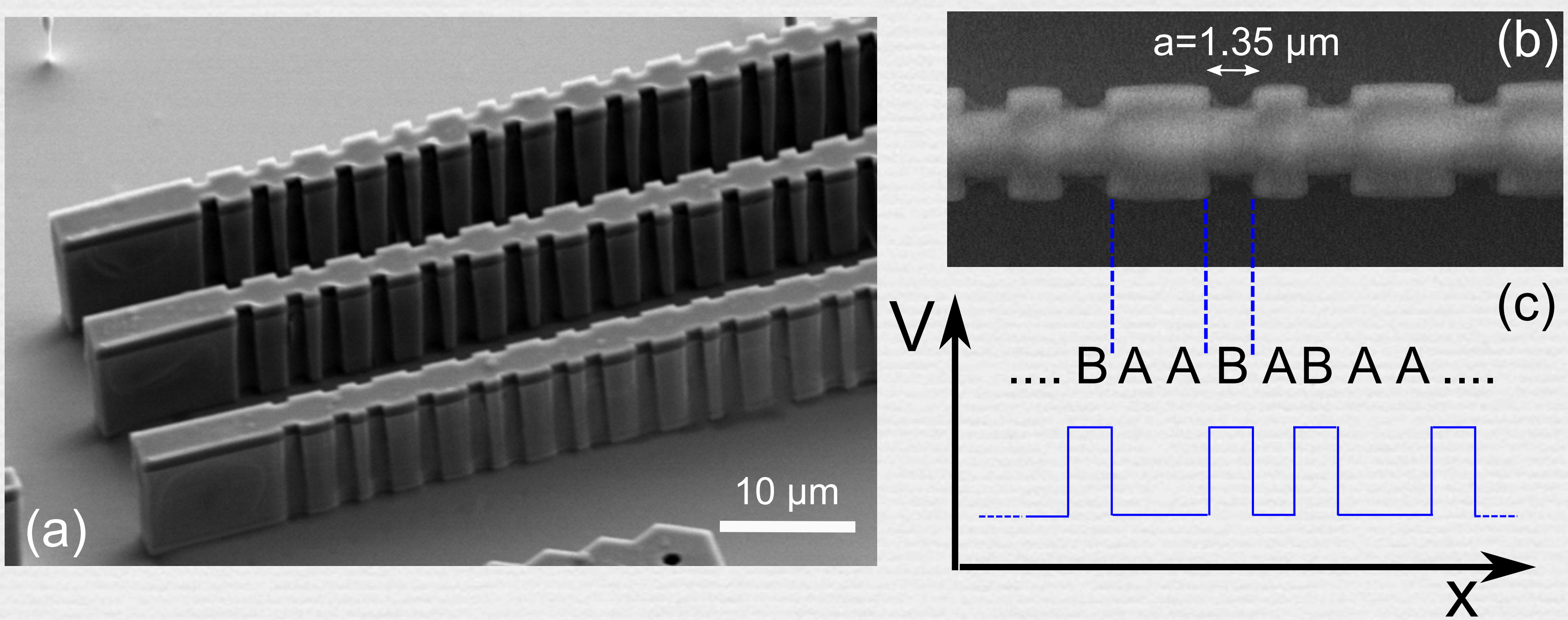
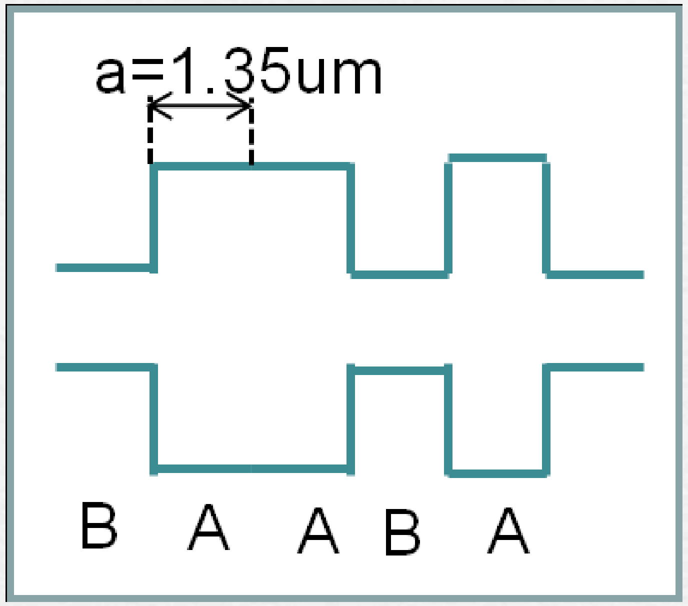
But still much to be done...



Number of letters of a sequence S_j is the Fibonacci number F_j so that $F_j = F_{j-1} + F_{j-2}$



(233 letters)



Fibonacci sequence: $S_{j\geq 2} = \left[S_{j-1} S_{j-2} \right], \quad S_0 = B, \quad S_1 = A$

A→AB→ABA→ABAAB→ABAABABA→...

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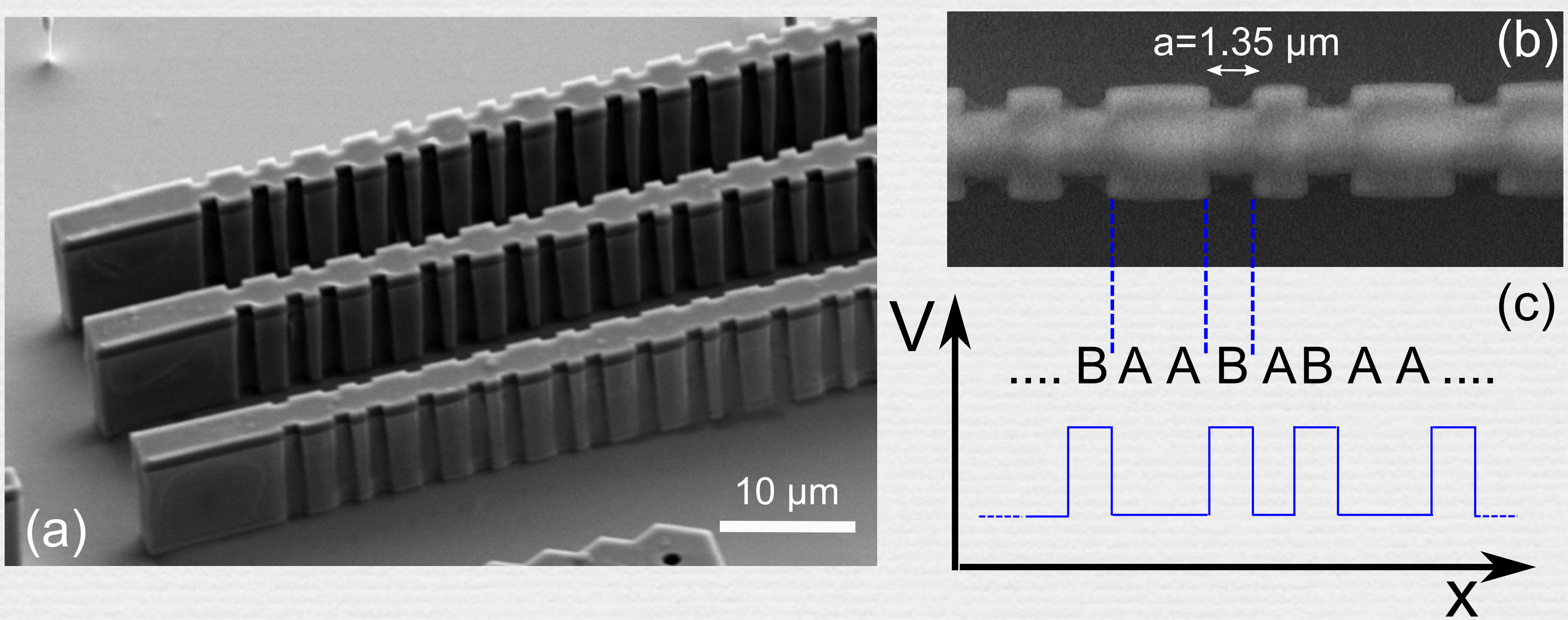

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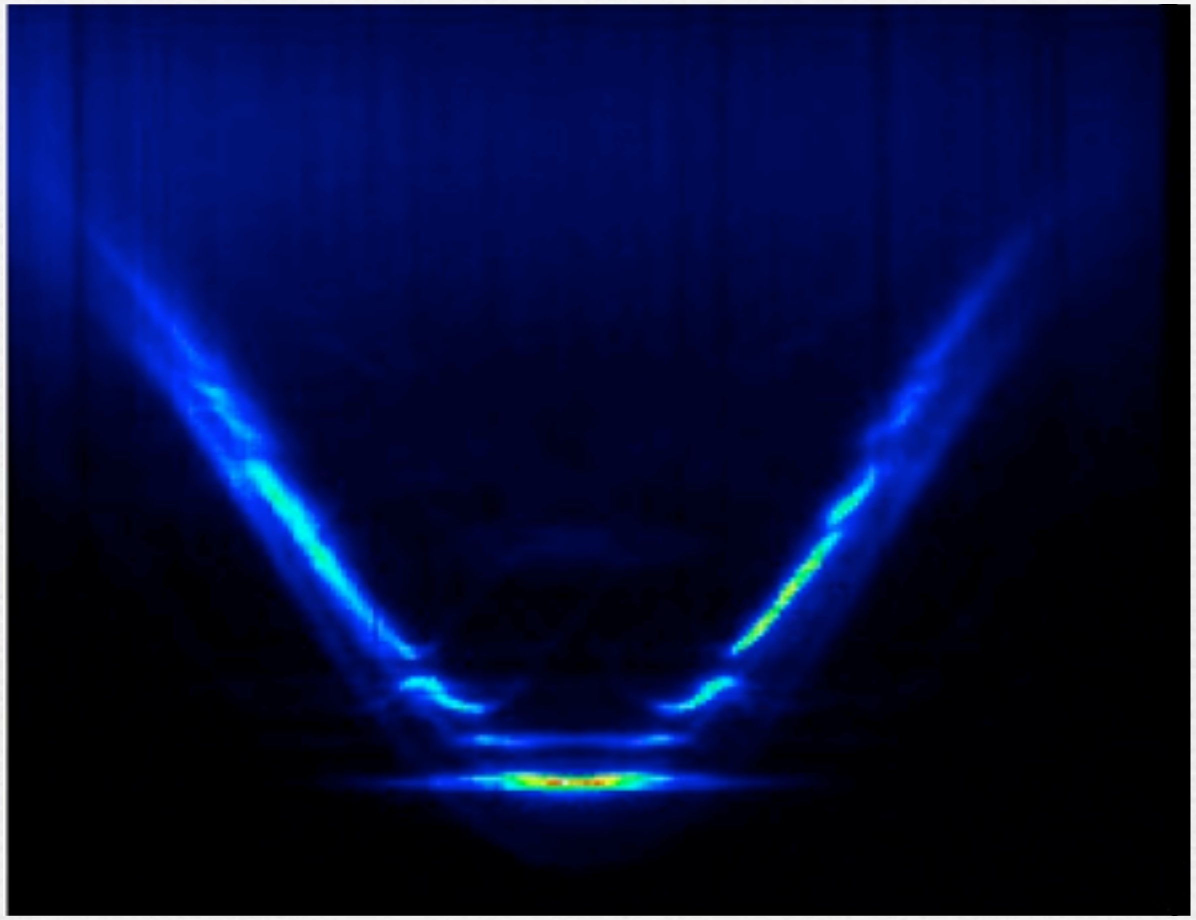
Measure of spectral function $E(k)$ intensity maps



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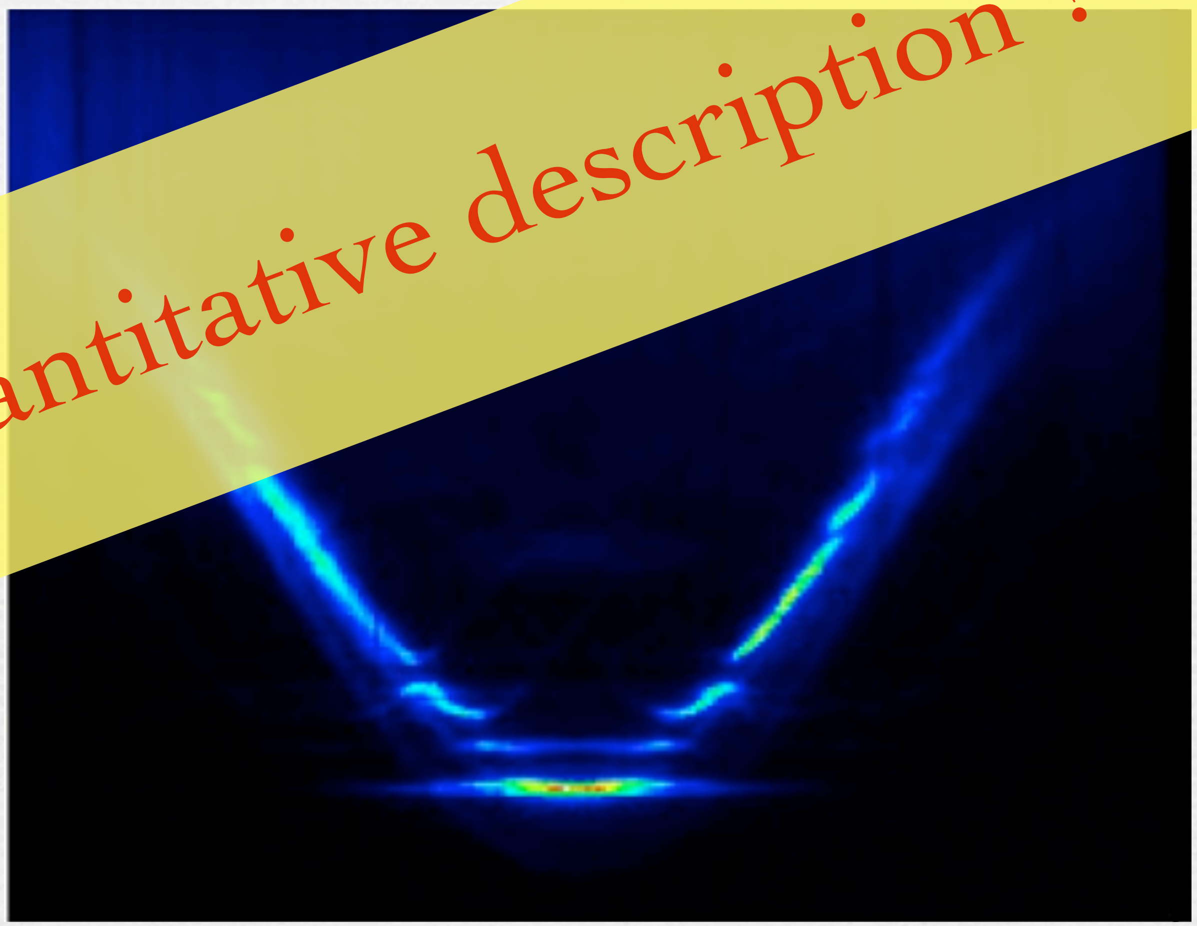


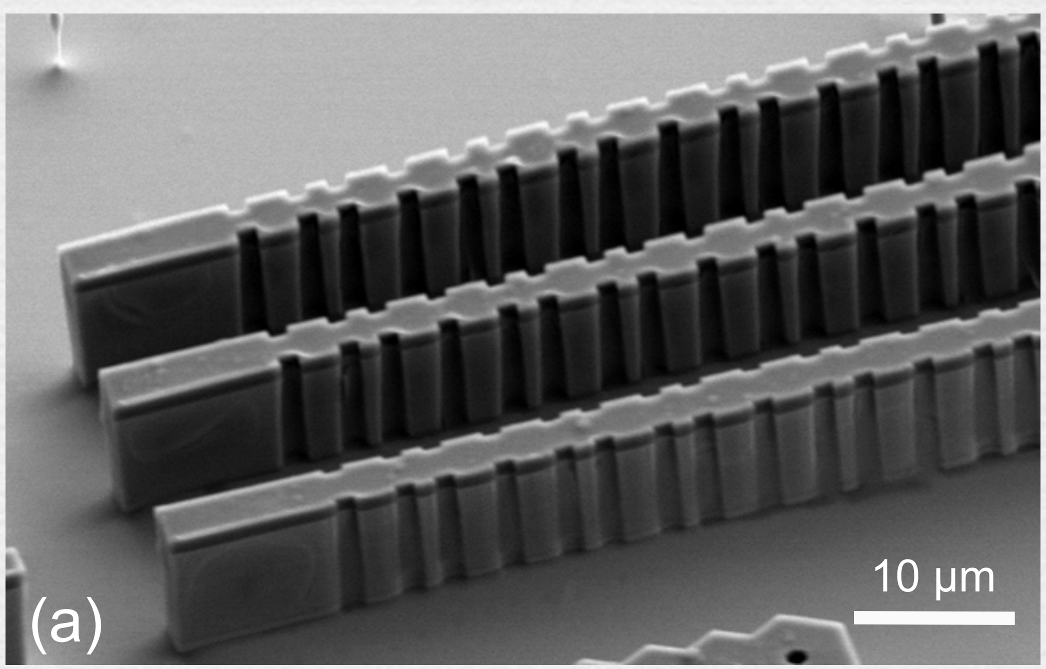
Quantitative description !

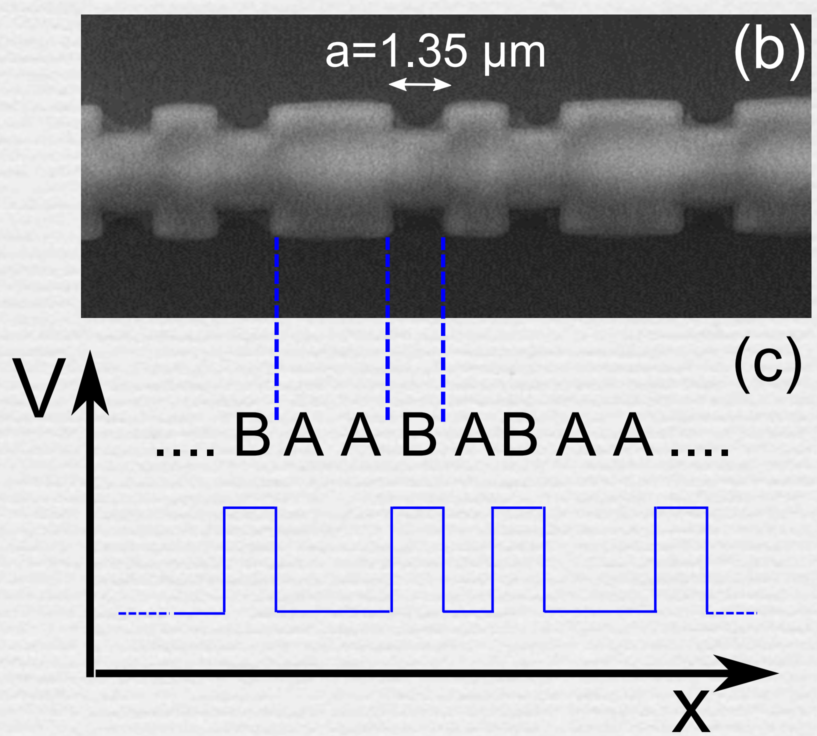
Effective 1D model

$$\left[-\frac{\hbar^2}{2M}\frac{d^2}{dx^2}+V(x)\right]\psi(x)=E\,\psi(x)$$

where

$$V(x)=\sum_n \chi\left(\sigma^{-1}n\right)u_b\left(x-an\right)$$





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$$\chi(x)=\begin{matrix}1, -\sigma<x<-\sigma^3\\ 0, -\sigma^3<x<\sigma^2\end{matrix}$$

$\sigma = \left(\sqrt{5}-1\right)\!/_2$ is the inverse golden mean

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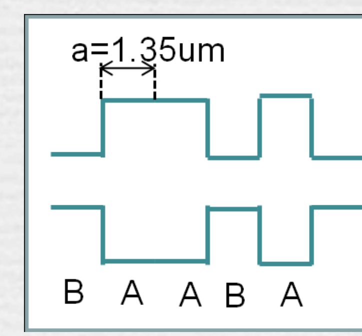
Shape of each letter



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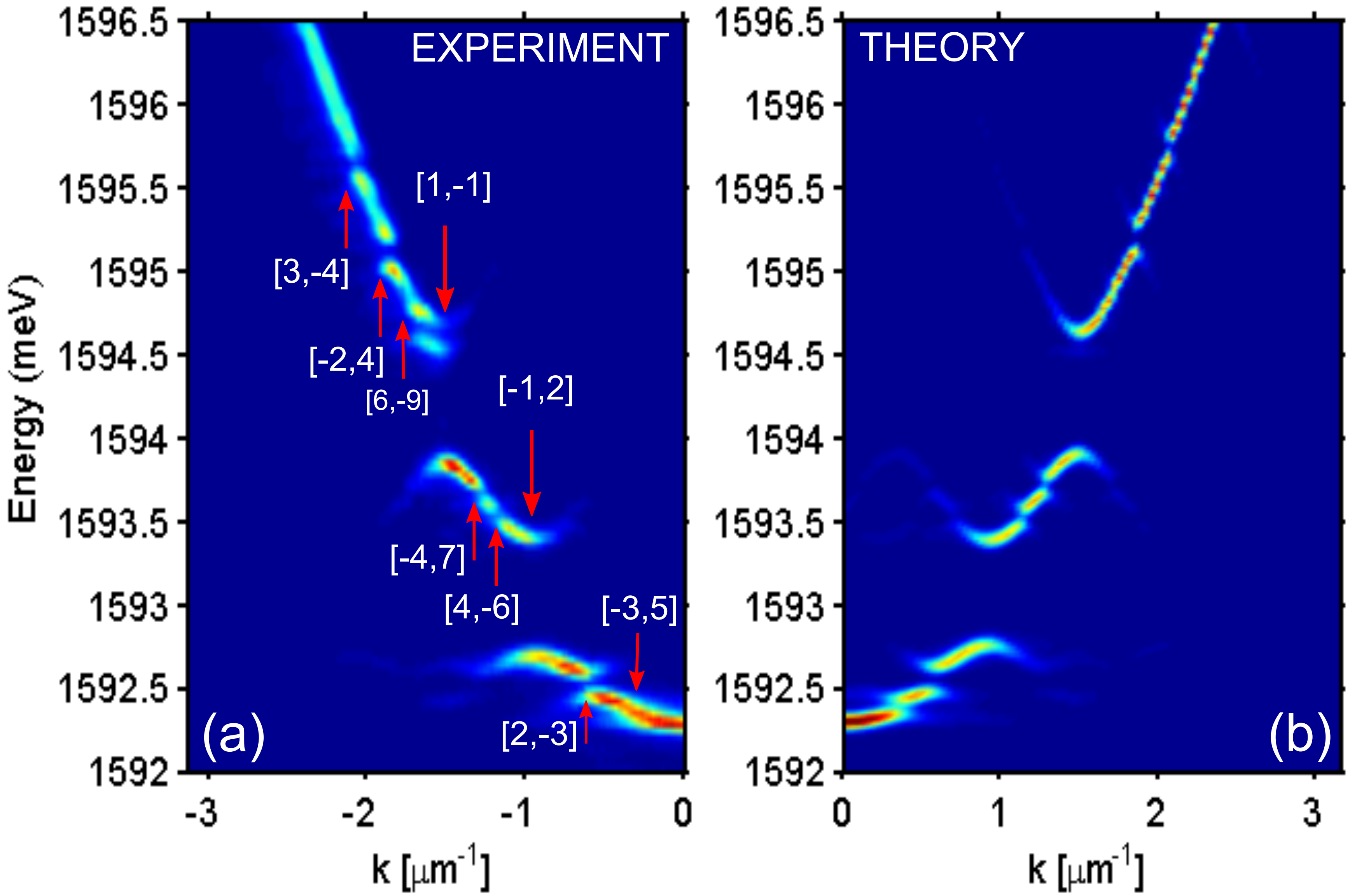
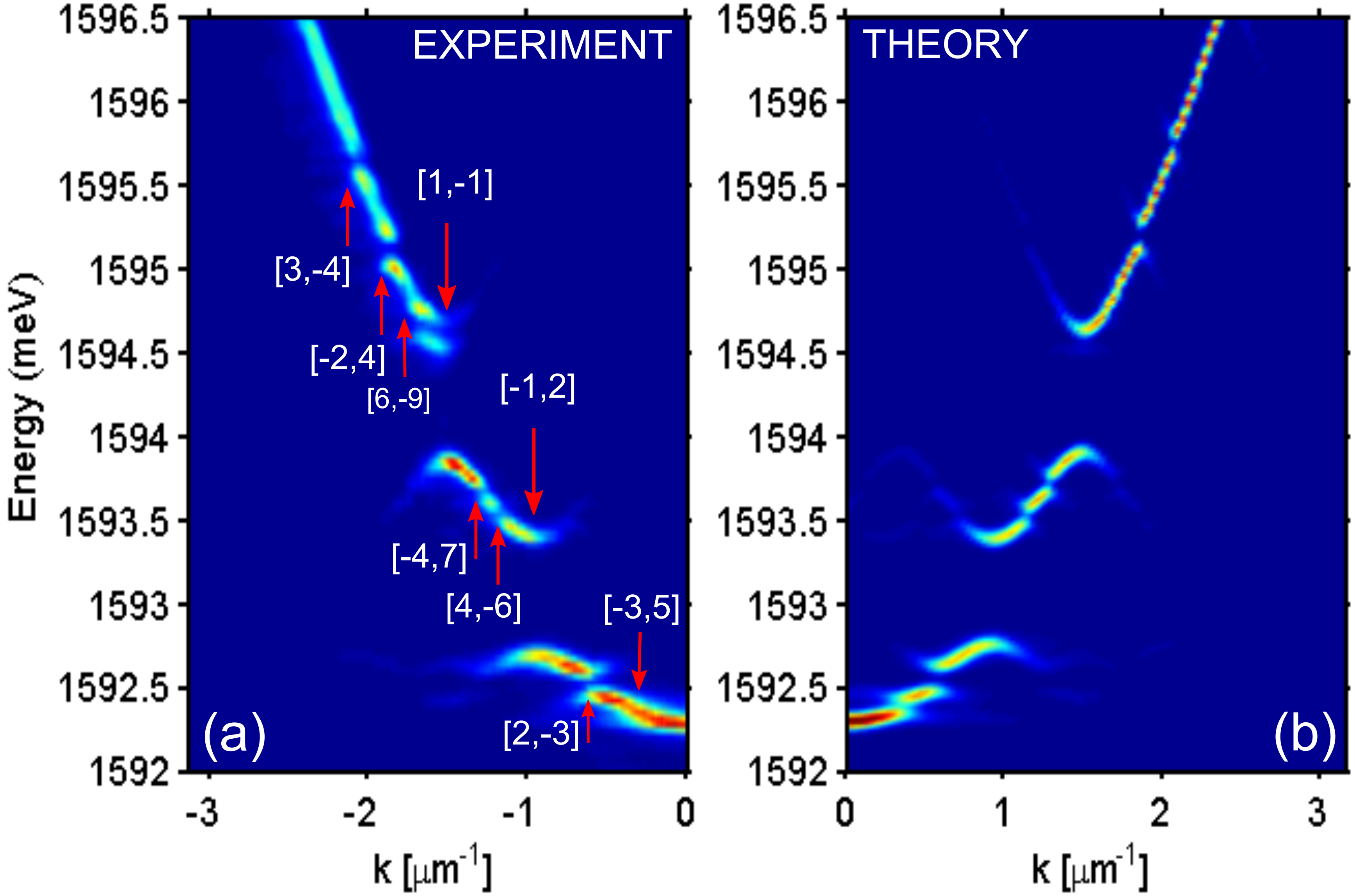
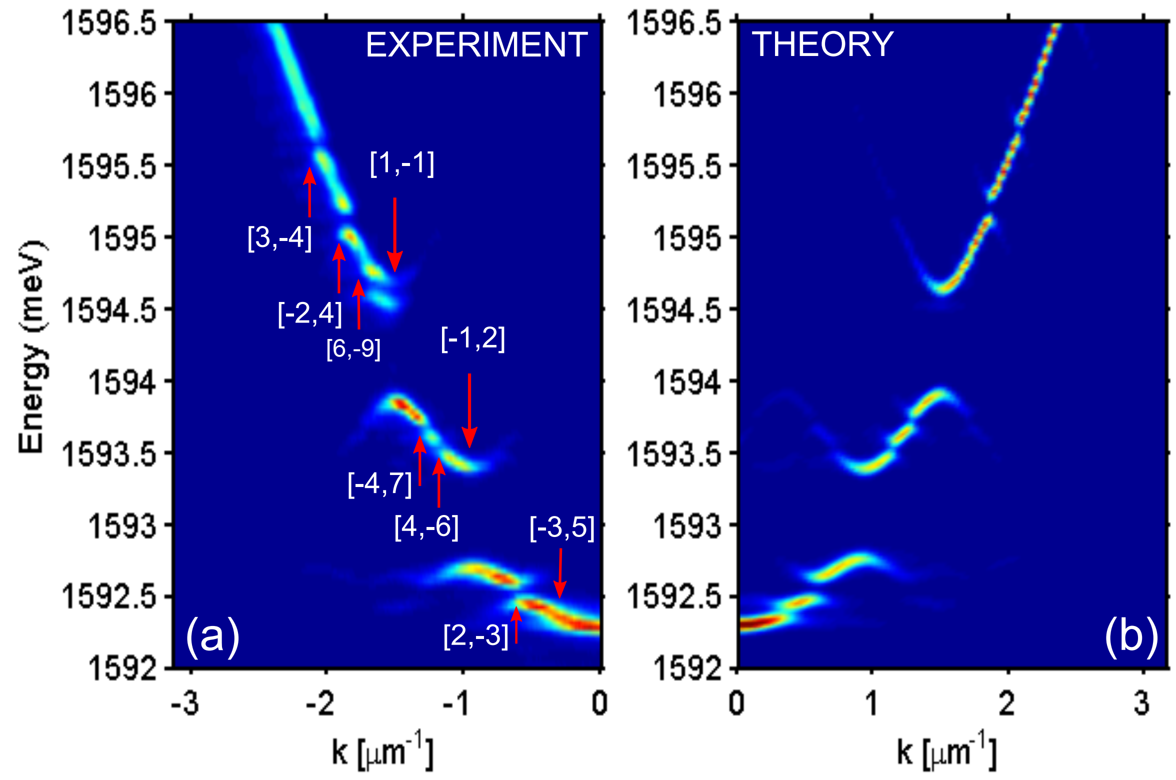

EXPERIMENT
THEORY
[1,-1]
[3,-4]
[-2,4]
[6,-9]
[-1,2]
[-4,7]
[4,-6]
[-3,5]
[2,-3]
(a)
(b)
Energy (meV)
1596.5
1596
1595.5
1595
1594.5
1594
1593.5
1593
1592.5
1592
-3
-2
-1
0
1
2
3
k [μm^{-1}]

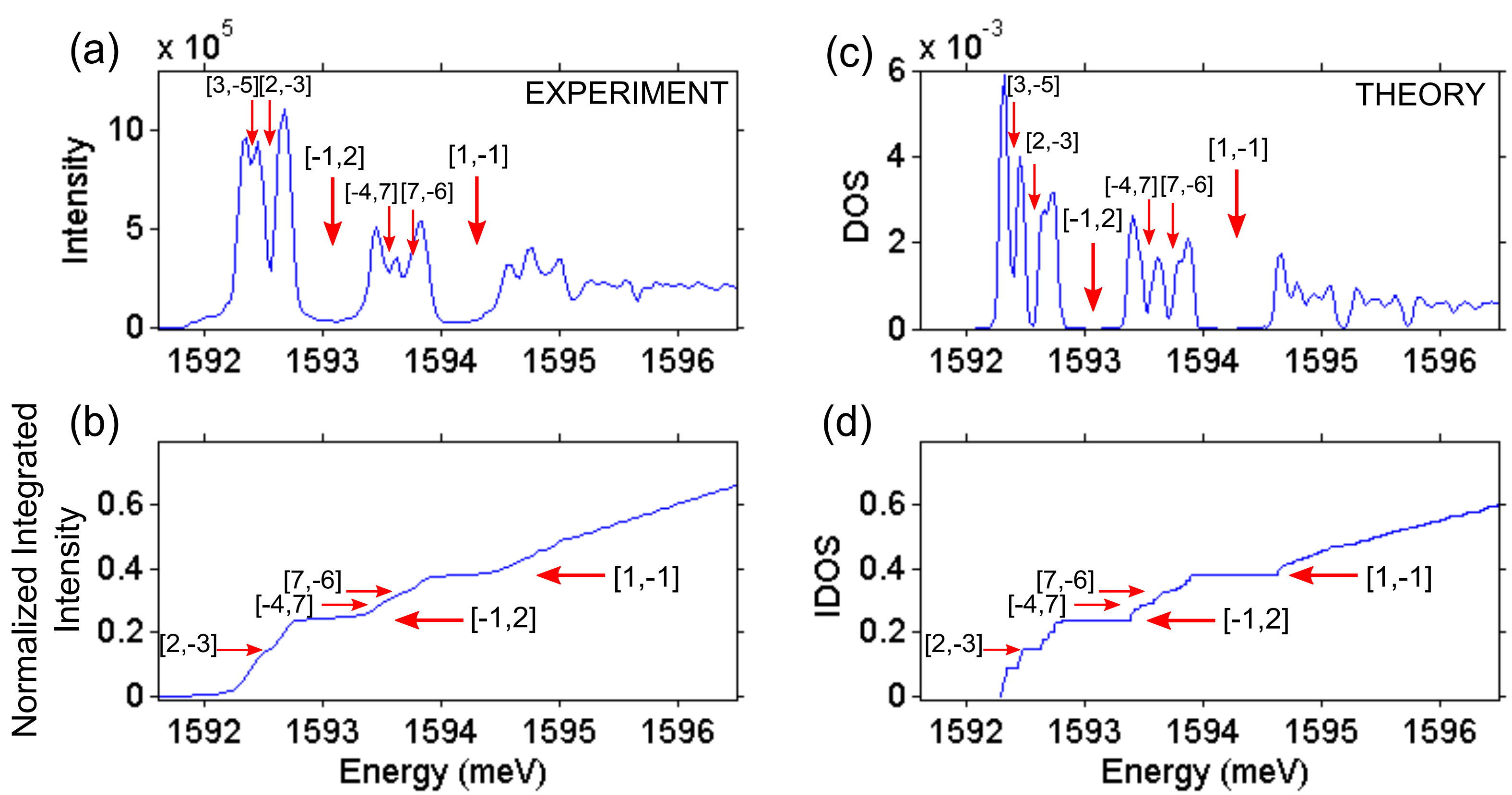
Labeling the gaps...



Labeling the gaps...



Calculating the integrated density of states (IDOS)



Integrated density of states (IDOS)-Gap labeling

$$\left[-\frac{\hbar^2}{2M}\frac{d^2}{dx^2}+V(x)\right]\psi(x)=E\,\psi(x)$$

where

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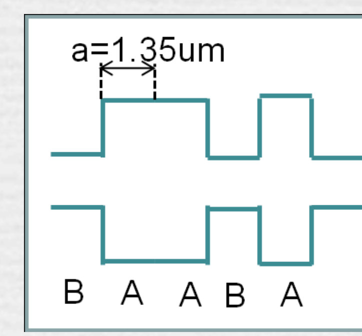

$\sigma=\left(\sqrt{5}-1\right)/2$ is the inverse golden mean

$$V(k) = u_b(k) \times \sum_{p,q} \chi_q \, \delta\left(ka - 2\pi\left[p + \sigma q\right]\right)$$

Each pair $\{p,q\}$ of integers defines a unique Bragg peak (σ is irrational).

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Bragg peaks are dense (Cantor set) $\Longrightarrow$ Must use periodic approximants, *i.e.* replacing irrational σ by

$$\sigma \approx F_j / F_{j+1}$$

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Periodic crystal of length $a\,F_{j+1}$ and potential

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Bragg peaks at values $k=Q\equiv\frac{1}{a}\left(F_{j+1}\,p+F_j\,q\right)\xrightarrow[j\to\infty]{}\frac{1}{a}\left(p+q\sigma\right)$

Perturbation theory

$$\left[-\frac{\hbar^2}{2M}\frac{d^2}{dx^2}+V(x)\right]\psi(x)=E\,\psi(x)$$

small

Perturbation theory

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small

Experimentally, it is not the case !

Perturbation theory (small V)

For the (quasi) crystal, a series of gaps open at each value of the (independent) Bragg peaks (Bloch thm.).

$$k = Q \equiv \frac{1}{a}\left(F_{j+1}\, p + F_j\, q\right) \xrightarrow[j\to\infty]{} \frac{1}{a}\left(p + q\,\sigma\right)$$

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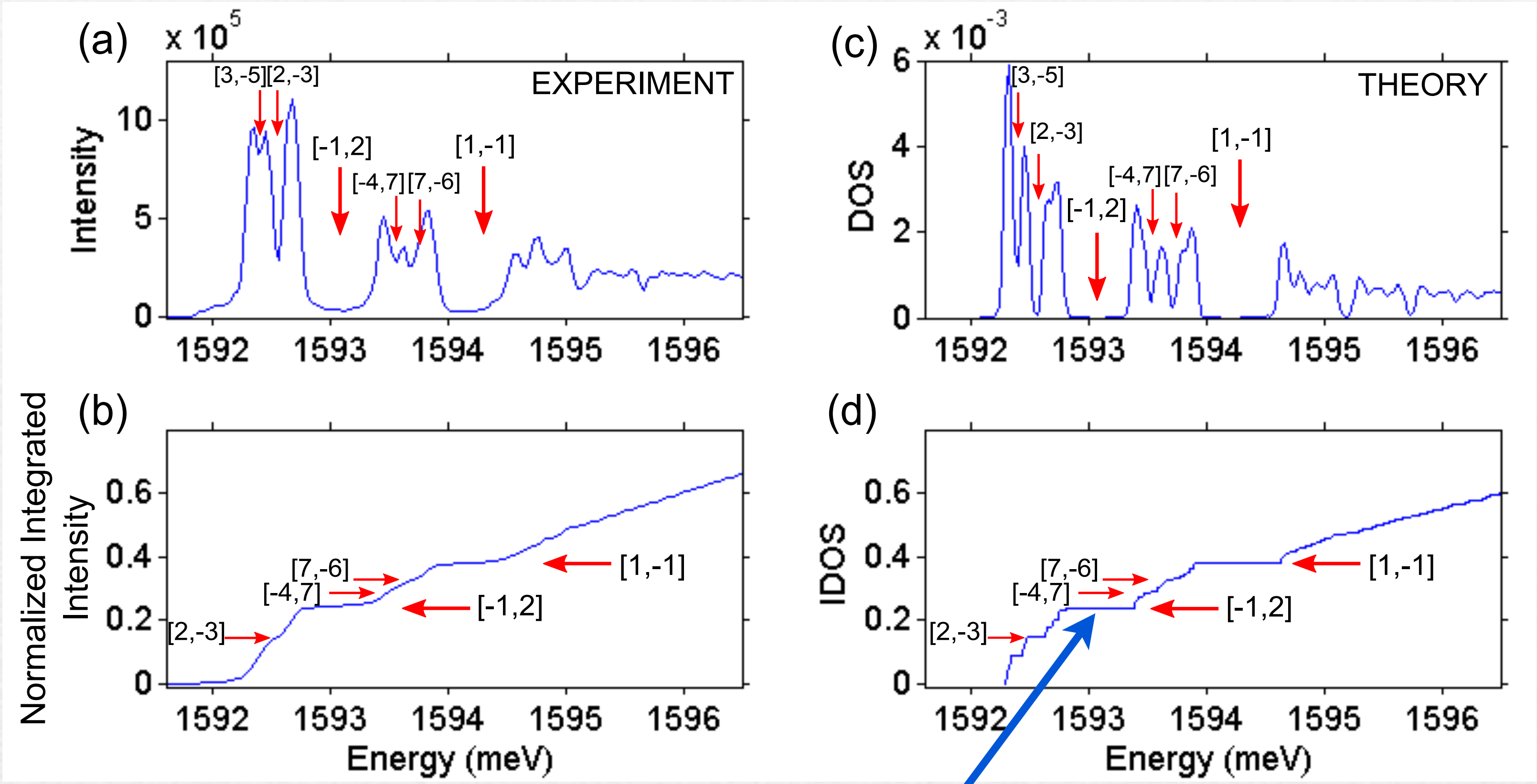
The (normalized) IDOS inside a gap labeled by $\{p,q\}$ is

$$N\left(\varepsilon = E_{Q_{p,q}/2}\right) = p + q\,\sigma$$

Integrated Density of States-Gap Labeling

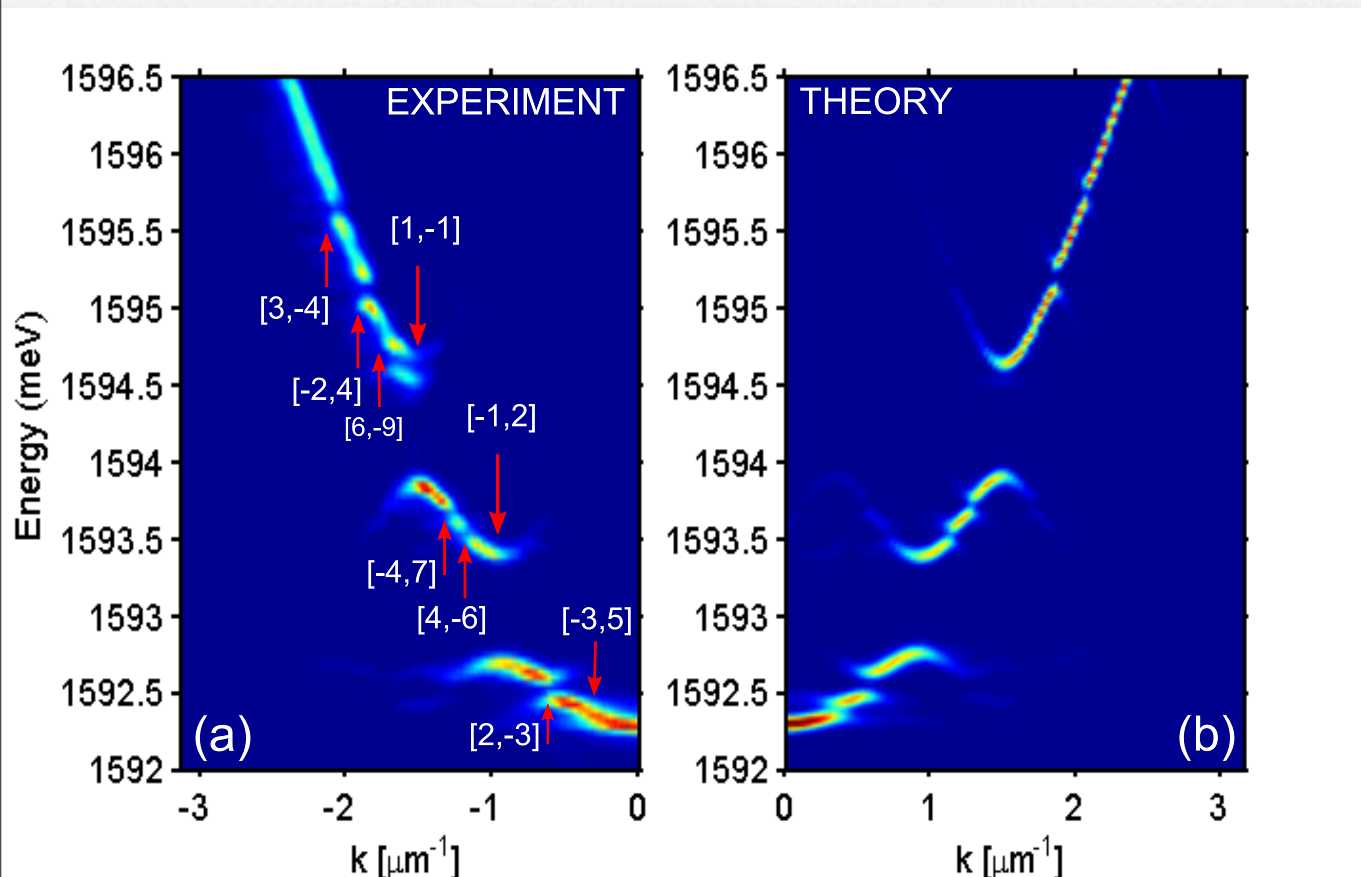


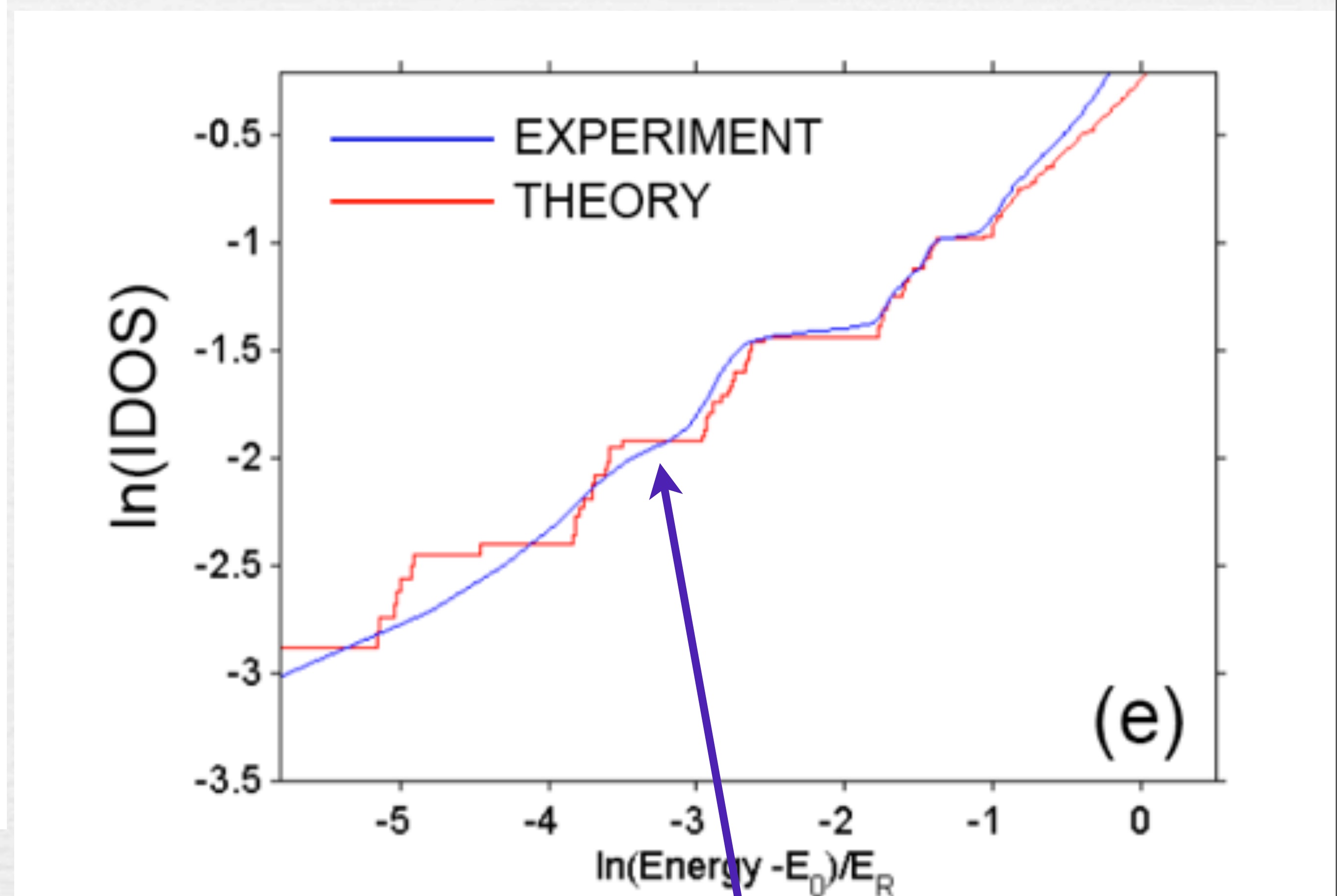
$$N\left(\varepsilon = E_{Q_{p,q}/2}\right) = p + q\sigma$$ within a $\{p,q\}$ gap

Topological invariants - independent of potential strength, inhomogeneity, ...

Integrated Density of States-Log-periodic oscillations

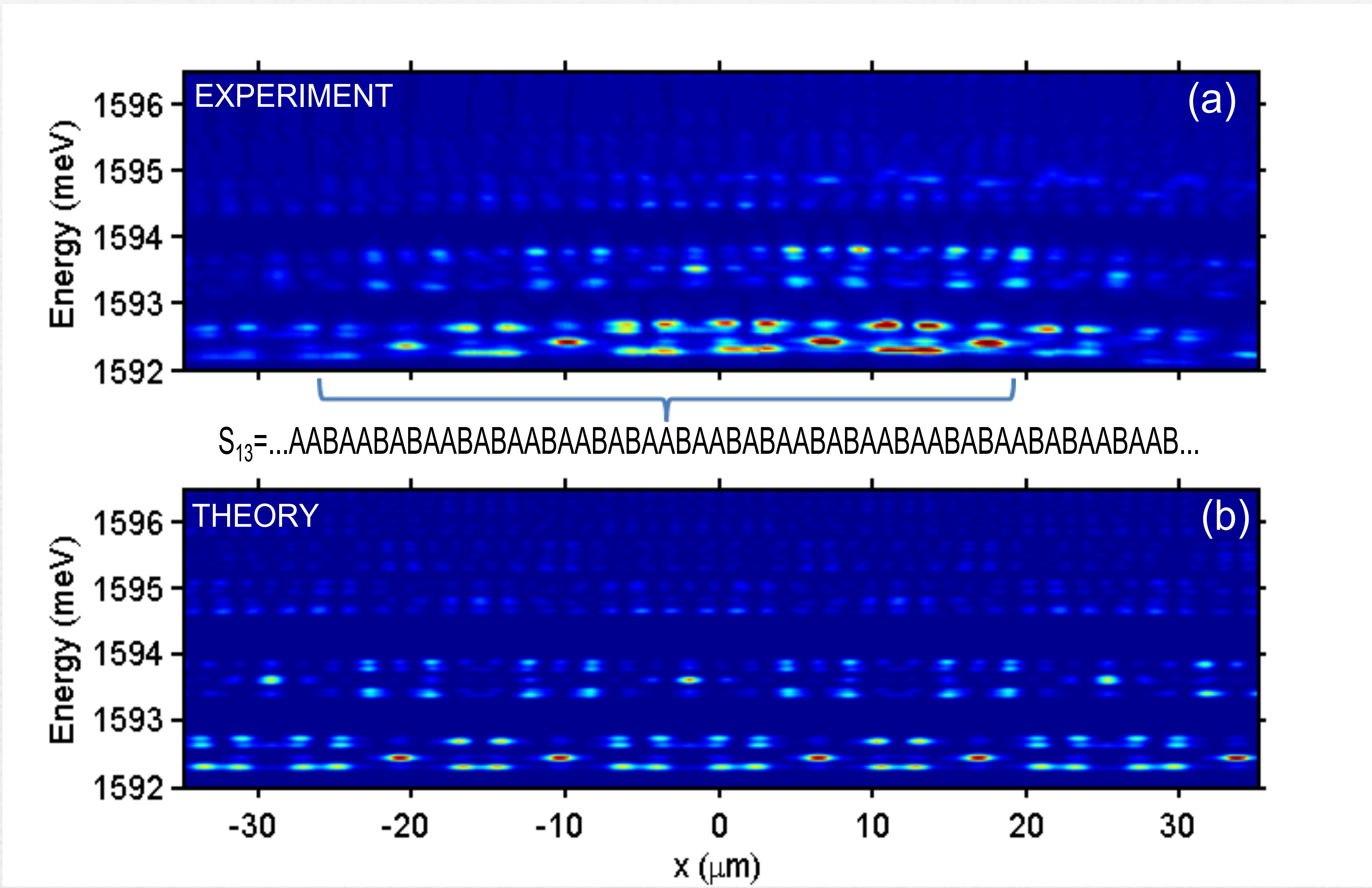
outside $\{p,q\}$ gaps





$$N_\omega(\Delta\omega) = (\Delta\omega)^\alpha \times F\left(\frac{\ln|\Delta\omega|}{\ln b}\right), \qquad \alpha = \frac{\ln a}{\ln b}, \quad F(x+1) = F(x)$$

Imaging the modes in real space : spatially and spectrally resolved emission



Summary-Further directions

- Coupling of a quantum emitter to a fractal quasi-continuum leads to an unusual decay dynamics.
- The decay exhibits scaling properties related to the discrete scaling symmetry of the quasi-continuum.
- The experimental study of a macroscopic coherent polariton gas in a Fibonacci cavity allows for a quantitative study of a fractal singular continuous energy spectrum : spectral function, wave functions and gap labeling.

Further directions

- Long time dynamics of wave packets with a quasi-continuum fractal spectrum. Log-periodic oscillations.

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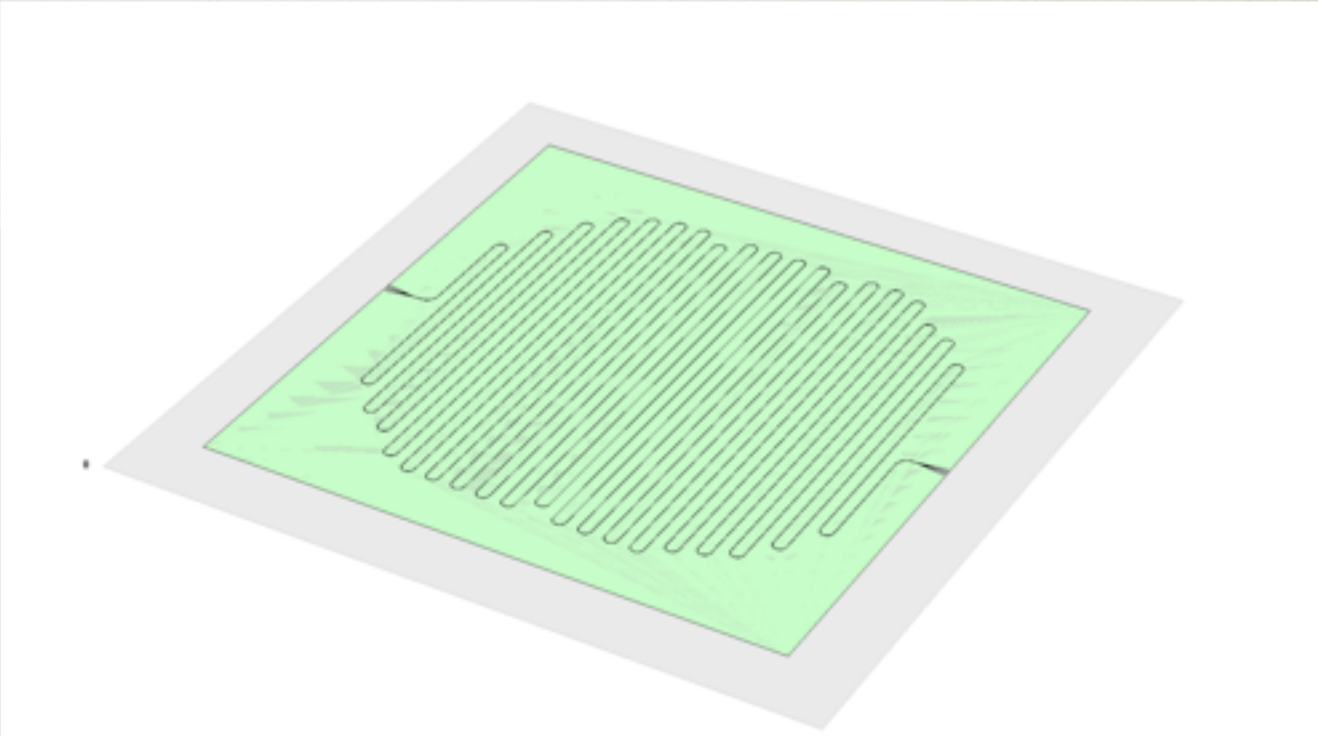

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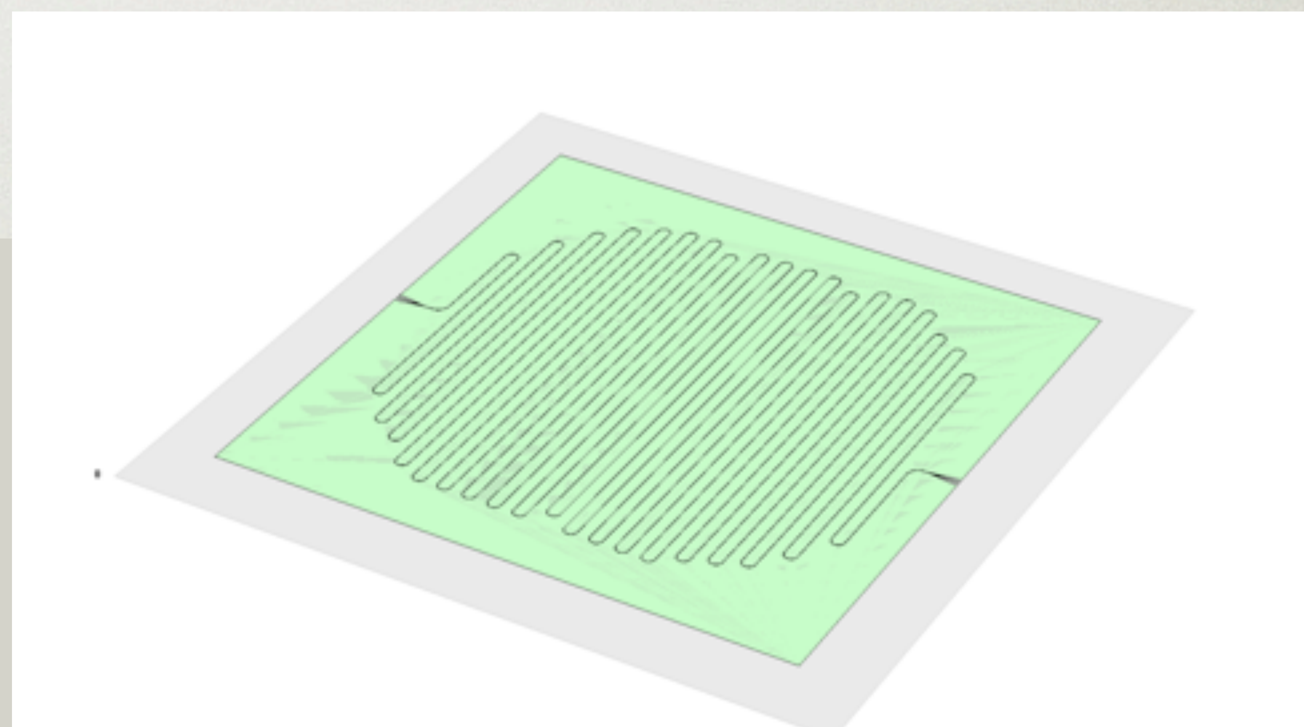
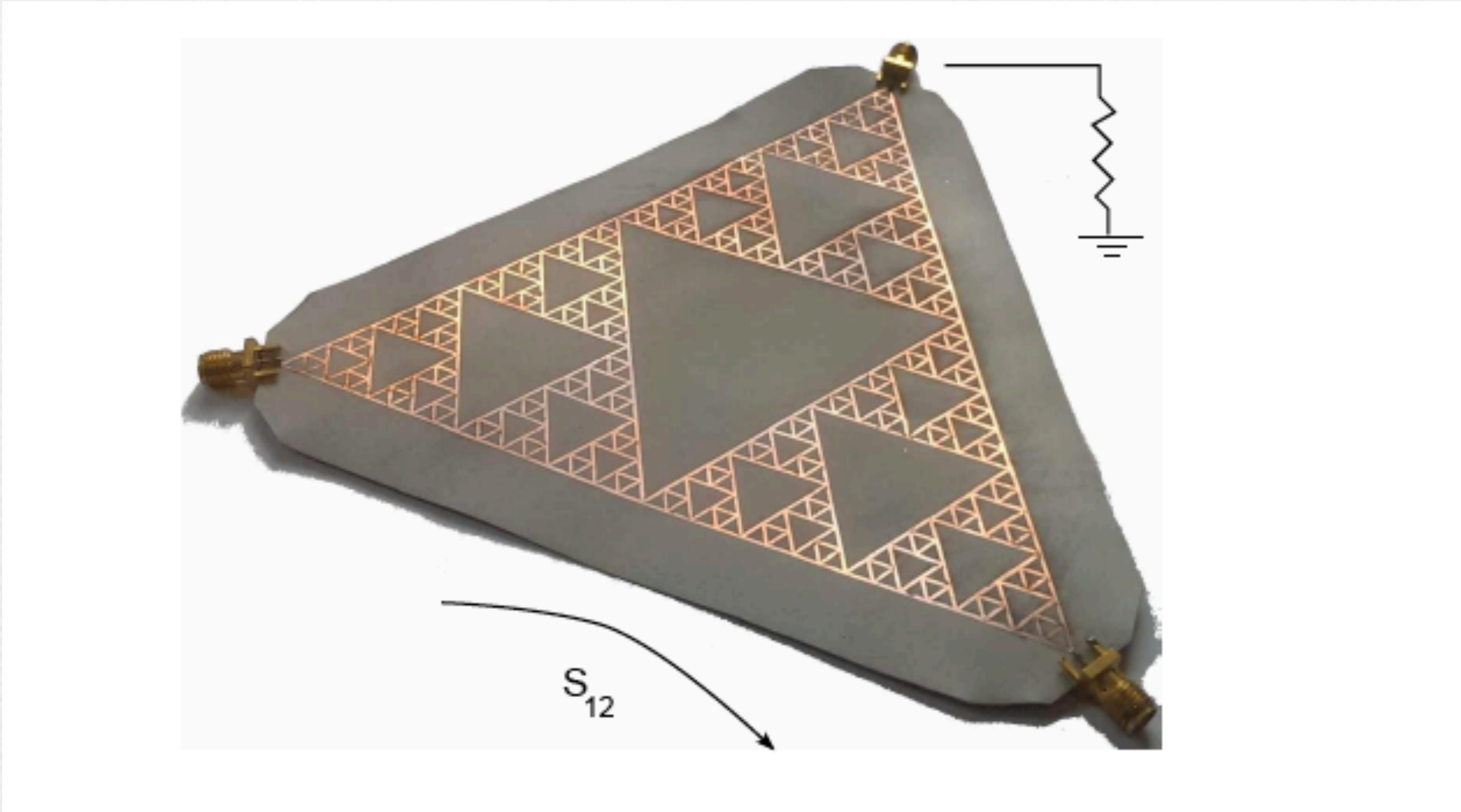
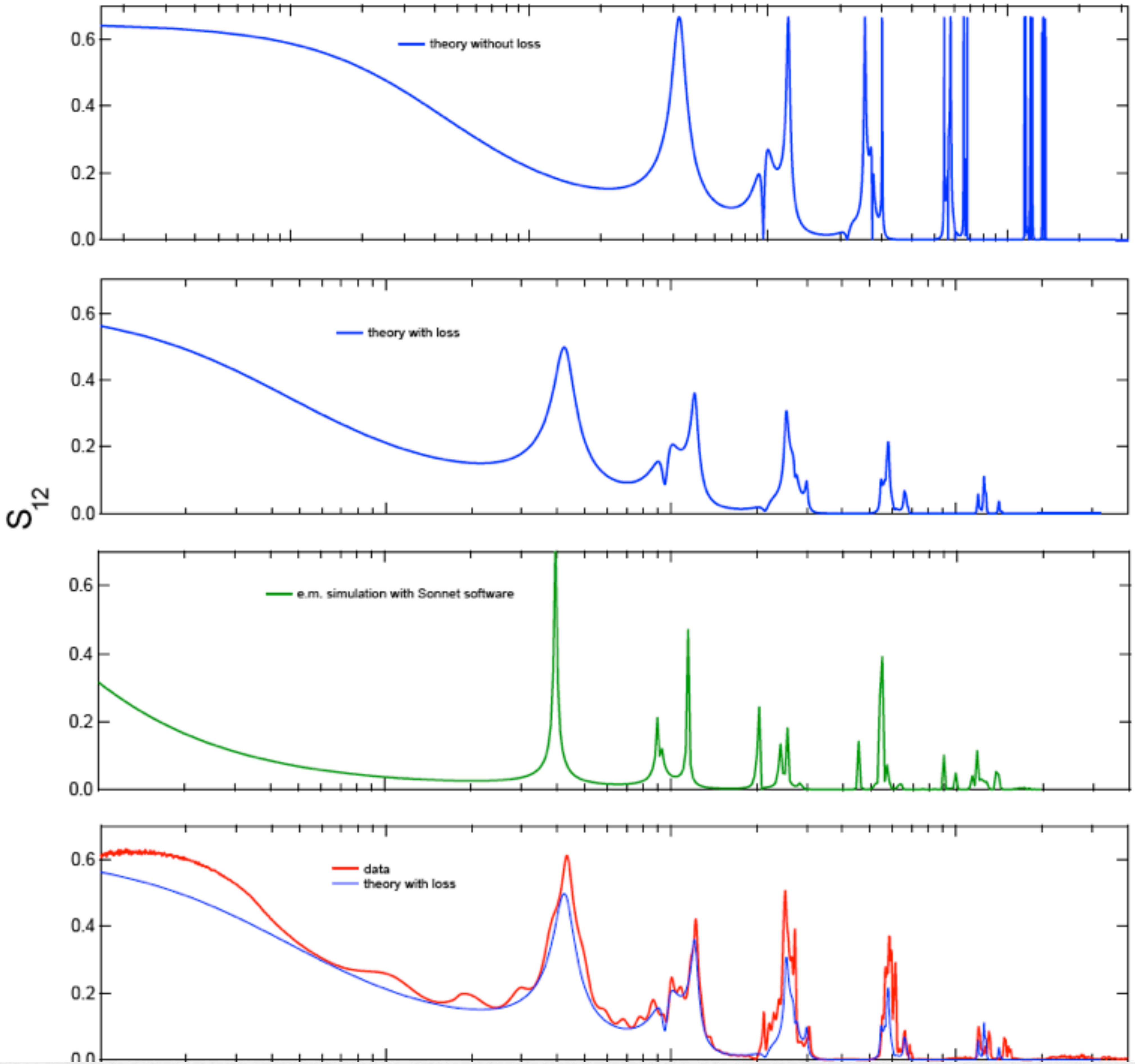


A preliminary experiment at room temperature





theory without loss
theory with loss
e.m. simulation with Sonnet software
data
theory with loss
S_{12}
0.0
0.2
0.4
0.6

Further directions -
Quantum gravity

- A hard problem ! Several approaches on the market.

Further directions - Quantum gravity

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- We consider quantum fluctuations around the classical (Einstein) solution (semi-classical approach)

Further directions - Quantum gravity

- Several approaches on the market.
- Consider here quantum fluctuations around the classical (Einstein) solution : semi-classical approach
- At short scales, the resulting space-time seems to have a fractal structure (Numerics & RG ⟶ Martin Reuter).

PRL 95, 171301 (2005) PHYSICAL REVIEW LETTERS

The Spectral Dimension of the Universe is Scale Dependent

J. Ambjørn,[1,3,*] J. Jurkiewicz,[2,†] and R. Loll[3,‡]
[1]The Niels Bohr Institute, Copenhagen University, Blegdamsvej 17, DK-2100 Copenhagen Ø,
[2]Mark Kac Complex Systems Research Centre, Marian Smoluchowski Institute of Physics, Jagelloni
Reymonta 4, PL 30-059 Krakow, Poland
[3]Institute for Theoretical Physics, Utrecht University, Leuvenlaan 4, NL-3584 CE Utrecht, The N
(Received 13 May 2005; published 20 October 2005)

We measure the spectral dimension of universes emerging from nonperturbative quantum defined through state sums of causal triangulated geometries. While four dimensional on large s quantum universe appears two dimensional at short distances. We conclude that quantum gravit "self-renormalizing" at the Planck scale, by virtue of a mechanism of dynamical dimensional r

DOI: 10.1103/PhysRevLett.95.171301 PACS numbers: 04.60.Gw, 04.60.Nc

Quantum gravity as an ultraviolet regulator?—A shared hope of researchers in otherwise disparate approaches to quantum gravity is that the microstructure of space and

tral dimension, a diffeomorphism-ir tained from studying diffusion on th of geometries. On large scales and v

Further directions - Quantum gravity

- At short scales, the resulting space-time seems to have a fractal structure.
- A good guess (See Sasha Teplyaev) seems to be a scalar quantum field on barycentric fractals.

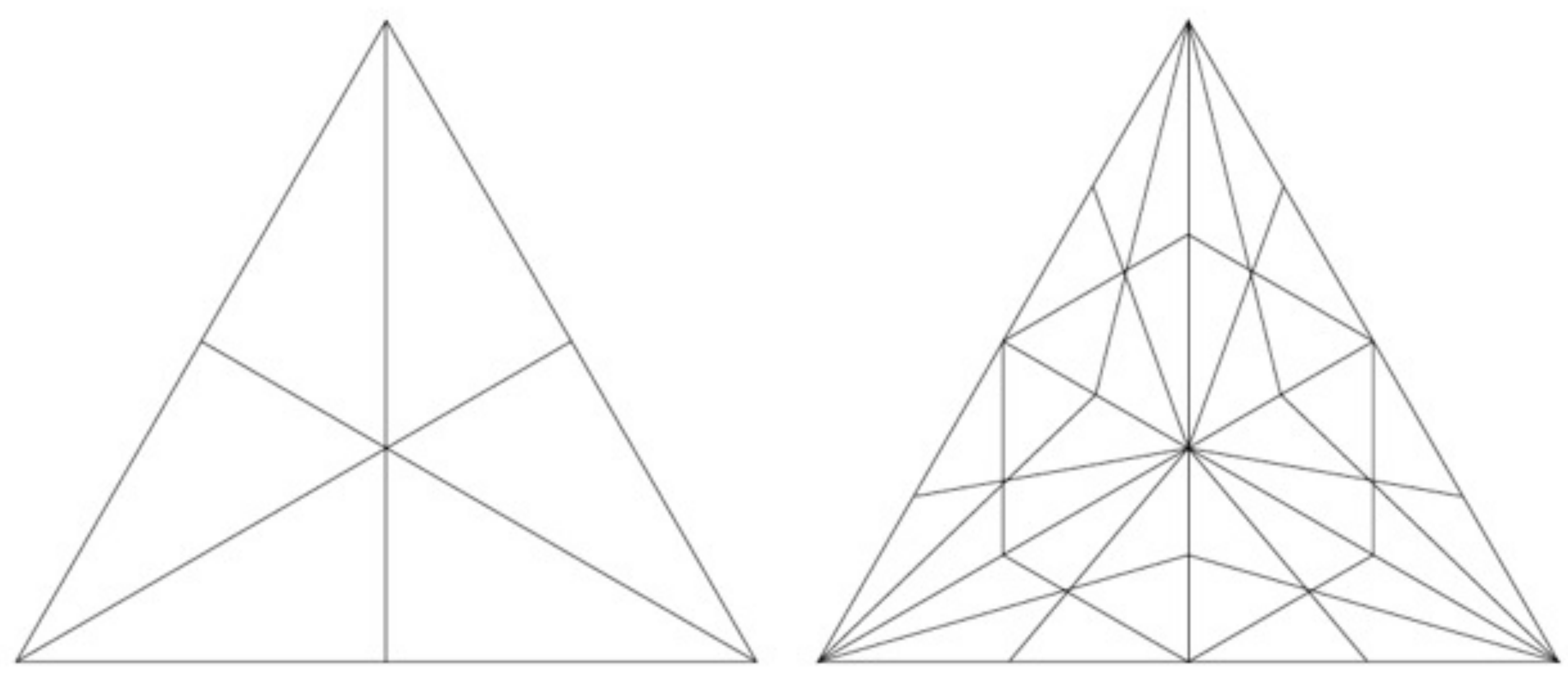

Figure 2.1. Barrycentric subdivision

Apparently not that weird...

Apparently not that weird...

F. Englert proposed a very similar idea back in 1986.

**METRIC SPACE-TIME AS FIXED POINT
OF THE RENORMALIZATION GROUP EQUATIONS
ON FRACTAL STRUCTURES**

F. Englert et al. / Metric space-time 177

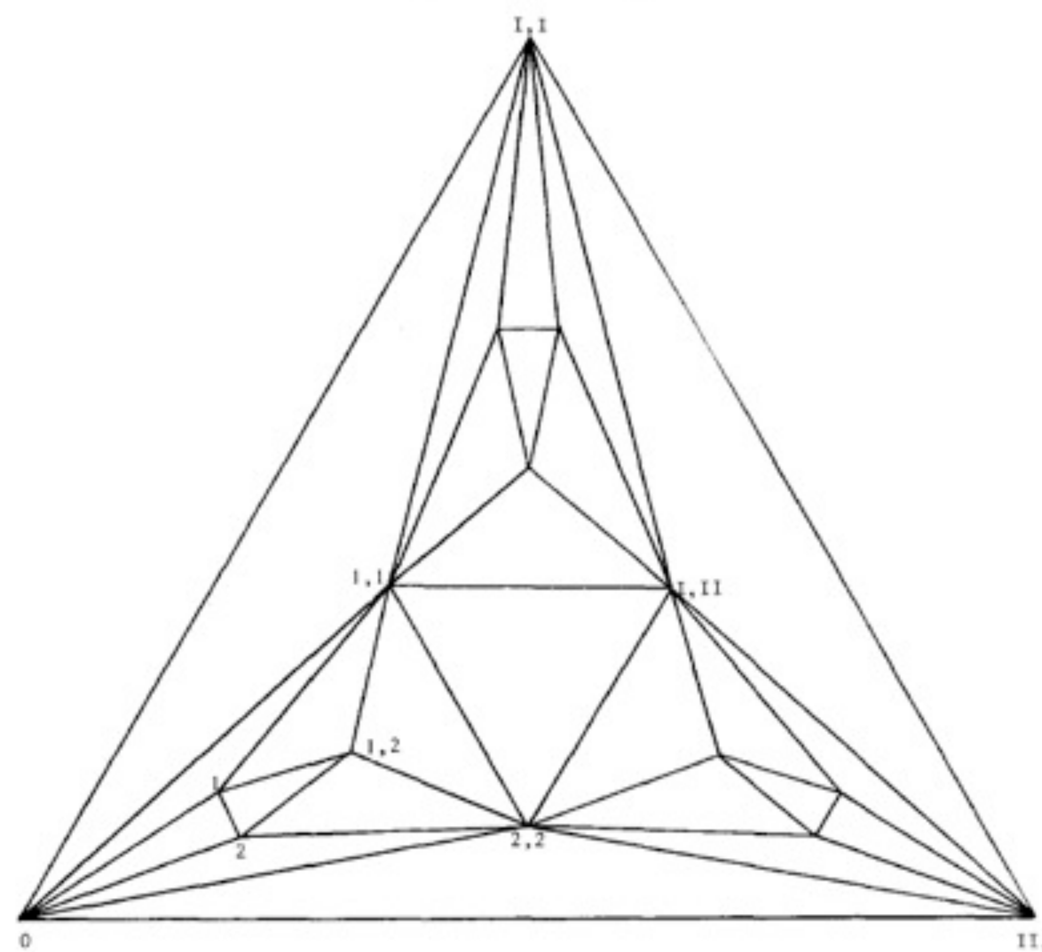

Fig. 10. A metrical representation of the two first iterations of a 2-dimensional 2-fractal corresponding to the euclidean fixed point. Vertices are labelled according to fig. 4.