Question 1. (13 points) The game of Hackenbush is played with one or several stick figures made of line segments with all segments connected through each other to a base line (the ground). A move consists of deleting any one line segment, which forces the deletion of any other line segment that is no longer connected to the ground. The last player to remove a segment loses.

(a) Draw the game tree for Hackenbush between 2 players, A (who goes first) and B, with the stick figure given in Figure 2 below. (At each turn, think about what edges are left for the player to remove.)

(b) Perform the game tree analysis (a.k.a. backward induction, a.k.a. pruning) to determine whether either player has a forced win, assuming best rational play.
Question 2. (13 points) Consider the Democratic primaries prior to the 2008 presidential election. Assume that Hillary Clinton and Barack Obama had a choice of waging an aggressive (negative) campaign directed at the other’s weaknesses, or waging a positive campaign based on their own strengths. Assume also that each felt that negative campaigning, unless answered in kind, would be advantageous to the one doing the negative campaigning, at least as far as the primaries are concerned. Notice, however, that mutual negative campaigning will certainly put the Democratic party in a worse position for the general election than mutual positive campaigning.

(a) Assuming that each candidate is more concerned with his or her own political success than doing what is best for the party, model this as a $2 \times 2$ game and discuss what this suggests as far as rational behavior on the part of the candidates. (Are there any dominant strategies?)

(b) How does your model change if we assume that each candidate has the party’s best interests in mind? What do you think would happen in this case?

Question 3. (13 points)

(a) Show that $(3, 3)$ is a non-myopic equilibrium in the theory of moves (i.e. sequential) version of Chicken, whose matrix is given below.

\[
\begin{array}{c|cc}
\text{Column} & C & N \\
\hline
C & (3, 3) & (2, 4) \\
N & (4, 2) & (1, 1) \\
\end{array}
\]

(b) Remembering that C stands for “swerve” and N stands for “not swerve”, does this result surprise you? Why or why not?

Question 4. (13 points)

(a) Analyze the dollar auction for $b = 3$ and $s = 3$, assuming both players play rationally and with the conservative convention.

(b) What does O’Neill’s theorem predict the best strategy will be for the dollar auction with the conservative convention for stakes of $\$2$ and bankrolls of $\$10$, if the auction is played with quarters?

Question 5. (8 points) Suppose that we have a voting system with voters A, B, C, D, in which a coalition is winning if and only if it contains an even number of voters (and is not empty). Is this a weighted voting system? Explain why or why not.
**Question 6. (20 points)** The four members of the superhero team the Fantastic Four are Reed (Mr. Fantastic), Sue (the Invisible Woman), Ben (the Thing), and Johnny (the Human Torch). To make decisions during their upcoming visit to the Negative Zone, they decide to use a weighted voting system with a quota of 8 votes and weights assigned according to the following table.

<table>
<thead>
<tr>
<th>Member:</th>
<th>Reed</th>
<th>Sue</th>
<th>Ben</th>
<th>Johnny</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight:</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Compute the Shapley-Shubik indices and the Banzhaf indices of all the members of the team. (You may use any method you like, but you must show all your work.)

**Question 7. (20 points)** Choose **FOUR of the seven** statements below. For each statement you chose, determine if it is true or false. If you answer “True” explain your answer, and if you answer “False” provide a counterexample.

(a) In the game of NIM starting with three piles of coins where each pile contains only 1 coin, the first player will always win.

(b) In every $2 \times 2$ ordinal game, there is at most one Nash equilibrium.

(c) In a $2 \times 2$ ordinal game, if CC gives the outcome $(4, 4)$ then C is a dominant strategy for both players.

(d) There are fewer than 1,000 different $2 \times 2$ ordinal games.

(e) Every swap robust yes–no voting system is weighted.

(f) Every trade robust yes–no voting system is swap robust.

(g) In a monotone yes–no voting system, if two voters are NOT equally desirable, then their desirability is incomparable.
Figure 3: From the webcomic **Abstruse Goose**