

Corrections to the book *Algebraic Topology* by Allen Hatcher

Some of these are more in the nature of clarifications than corrections. Most of the corrections have already been incorporated into later printings of the book and into the online version of the book.

- Table of Contents. In Chapter 1 the item “Applications to Cell Complexes” is on page 49 rather than 50, as of late 2015.
- Chapter 0, page 9, line 12. Change “lines” to “line”.
- Chapter 0, page 9. In the next-to-last paragraph delete the sentence “This viewpoint makes it easy to see that the join operation is associative.” Also, in the sentence preceding this one, change the word “regarded” to “constructed”. Set-theoretically it is true that join is associative, but there are examples where the topologies on $(X * Y) * Z$ and $X * (Y * Z)$ can be different. This is another instance of how mixing product and quotient constructions can lead to bad point-set topological behavior. For CW complexes the issue can be avoided by using CW topologies, as in the first paragraph at the top of the next page.
- Chapter 0, page 9, line -11. Replace $0t_i$ by $0x_i$.
- Chapter 0, page 14. The discussion of the homotopy extension property in the middle of this page skims over a somewhat delicate question in point-set topology, whether a function $X \times \{0\} \cup A \times I \rightarrow Y$ is continuous if its restrictions to $X \times \{0\}$ and $A \times I$ are continuous. This is true if A is closed in X , which covers most applications of the homotopy extension property. The online version of the book gives a corrected version of the argument. The trickier case that A is not assumed to be closed has been added to the Appendix.
- Chapter 0, page 15, Example 0.15. If you have an early version of this chapter with no figure for this example, then in the next-to-last line of this paragraph change “the closure of $X - N$ ” to “ $X - h(M_f - Z)$ ”. [This paragraph was rewritten for later versions, making this correction irrelevant.]
- Chapter 0, page 16. The second sentence of the proof of Proposition 0.18 may need further explanation of the underlying point-set topology. The assertion is that $X_0 \sqcup_f X_1$ and $X_0 \sqcup_g X_1$ are subspaces of $X_0 \sqcup_F (X_1 \times I)$, so in particular the quotient topologies on $X_0 \sqcup_f X_1$ and $X_0 \sqcup_g X_1$ are the same as the subspace topologies from $X_0 \sqcup_F (X_1 \times I)$. This is a special case of the following general fact about quotient topologies. Suppose $q: X \rightarrow Y$ is a quotient map, so q is surjective and a set V in Y is open if and only if $q^{-1}(V)$ is open in X . For a subspace B of Y one can ask whether the restriction map $q: A \rightarrow B$ from the subspace $A = q^{-1}(B)$ to B is also a quotient map. This is not always the case, but it is true if A is closed in X . To see this, let us define a subset U of X to be *saturated* if $U = q^{-1}(q(U))$. What must be checked

is that the saturated open sets in the subspace A are the intersections of saturated open sets in X with A . Obviously the intersection of a saturated open set in X with A is an open set in A that is saturated. Conversely, if A is closed in X and U is an open set in X such that $A \cap U$ is saturated in A , then $U \cup (X - A)$ is a saturated open set in X whose intersection with A is $A \cap U$.

■ Chapter 0, page 17. The fourth line should say that (Y, A) has the homotopy extension property, rather than (X, A) . Also, in the next paragraph there are two places where $k_{tu} : A \rightarrow A$ should be changed to $k_{tu} : A \rightarrow X$, in the fourth and twelfth lines following the displayed formula for k_t .

■ Chapter 0, page 17. In the third-to-last line, $f_1 : X \rightarrow X$ should be $f_1 : X \rightarrow Y$. Also, on the seventh-to-last line it might be clearer to say “Viewing k_{tu} as a homotopy of $k_t|_A$ ”

■ Chapter 0, page 19, Exercise 21. The space X should be assumed to be Hausdorff. For a more general version, let X be a connected quotient space of a finite set of disjoint 2-spheres obtained by identifying finitely many finite sets of points.

■ Chapter 0, page 20, Exercise 26, third line: Change (X, A) to (X_1, A) .

■ Chapter 0, page 20, Exercise 27. To avoid point-set topology difficulties, assume that f is not just surjective but a quotient map. Here is a more general version of this exercise: Given a pair (X, A) and a homotopy equivalence $f : A \rightarrow B$, show that the natural map $X \rightarrow B \sqcup_f X$ is a homotopy equivalence if (X, A) satisfies the homotopy extension property.

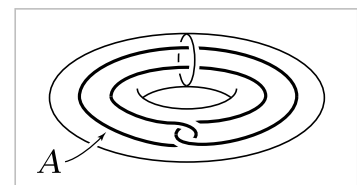
■ §1.1, page 30, line 14. Change “paths lifting the constant path at x_0 ” to “paths lifting constant paths”

■ §1.1, page 32, third paragraph. The reference should be to Corollary 2.15, not 2.11.

■ §1.1, page 32, last paragraph. The reference should be to Corollary 2B.7, not Proposition 2B.6.

■ §1.1, page 36, line 6. The reference should be to Theorem 2.26, not 2.19.

■ §1.1, page 39, Exercise 16(c). In case it’s not clear, the circle A is supposed to be the dark one in the figure, in the interior of the solid torus.



■ §1.2, page 46, sixth line from bottom. Repeated “the” — delete one.

■ §1.2, page 53, Exercise 5. Part (b) is simply wrong, and should be deleted.

■ §1.2, page 53, Exercise 6. Add the assumption that the discrete subspace of \mathbb{R}^n is closed. The result still holds without this condition but the proof is considerably more complicated.

■ §1.2, page 54, Exercise 15. It should be specified that if the triangle T has vertices P, Q, R , then the three edges are oriented as PQ, PR, QR .

- §1.2, page 55, line 1. A comment: the reduced suspension depends on the choice of basepoint, so the statement is that C is the reduced suspension of CX with respect to a suitable choice of basepoint.
- §1.3, page 56, second paragraph. A comment about the definition of a covering space: The way that $p^{-1}(U_\alpha)$ could be empty is that it could be the union of an empty collection of open sets homeomorphic to U_α .
- §1.3, page 57, third-to-last line. Change Koenig to König, to agree with the spelling in the Bibliography and in the original source itself.
- §1.3, page 61, next-to-last line of the proof of Proposition 1.32: $g_1\bar{g}_2$ should be $g_1 \cdot \bar{g}_2$, with a dot to denote composition of paths.
- §1.3, page 63. Typo in the next-to-last line of the third-to-last paragraph: “simply-connected” has two n’s, not three.
- §1.3, page 65, line 12. Change “cover space” to “covering space”
- §1.3, page 69, second and third lines of last paragraph. It should say “assuming that X is path-connected, locally path-connected, and semilocally simply-connected”.
- §1.3, page 79, Exercise 3. Add the hypothesis that the covering space map $p: \tilde{X} \rightarrow X$ is surjective.
- §1.3, page 79, Exercise 8. The reference should be to Exercise 11 in Chapter 0, not Exercise 10.
- §1.3, page 82, Exercise 33. Change the ℓ in the fourth line to d .
- §1.B, page 94. In the middle of the page, change the sentence that begins “To see this” so that it reads “To see this, note that $p: \tilde{A} \rightarrow A$ is a covering space, so we have injective maps $\pi_1(\tilde{A}) \rightarrow \pi_1(A) \rightarrow \pi_1(X)$ whose composition factors through $\pi_1(\tilde{X}) = 0$, hence $\pi_1(\tilde{A}) = 0$.”
- §1.B, page 94, seventh line up from the bottom. Change “component \tilde{A} of A ” to “component \tilde{A} of $p^{-1}(A)$ ”.
- §1.B, page 96, Exercise 9. Add the hypothesis that all the edge homomorphisms are injective.
- §2.1, page 109, line 5. The phrase “exactly two” is not quite correct since the two faces in a canceling pair could be faces of the same simplex. To fix this, replace this sentence by the following sentence: “If ξ is a cycle, all the $(n - 1)$ -dimensional faces of the Δ_i^n ’s are identified in pairs.” The online version of the book also contains some slight rewordings in the remainder of this paragraph, for the sake of clarity.
- §2.1, page 112. [Revised the text in the first paragraph to describe the subdivision of $\Delta^n \times I$ more geometrically. Also revised the beginning of the next paragraph and added a few more words on the next page in the paragraph following Proposition 2.12.]

- §2.1, page 120, line –12. Change w_i and w_j to w_j and w_k .
- §2.1, page 121, line –9. The equations should read $S([w_0]) = w_0(S\partial[w_0]) = w_0(S([\emptyset])) = w_0([\emptyset]) = [w_0]$. [This is corrected in later printings.]
- §2.1, page 123, line –3. Add a period at the end of this line.
- §2.1, page 125, Example 2.23. In the first paragraph of this example the sentence beginning “The second isomorphism” needs to be modified in the special case $n = 1$ since $\partial\Delta^{n-1}$ is empty in this case, which means that $(\Delta^{n-1}, \partial\Delta^{n-1})$ is not a good pair when $n = 1$. However the claimed isomorphism is easy to see in this case since it involves just H_0 . The online version of the book has been rephrased to deal with this issue. (The old version of this paragraph has 7 lines after the displayed formulas, the revised version has 8 lines.)
- §2.1, page 125, Example 2.23. Each occurrence of $H_n(S^n)$ in this example should have a tilde over the H .
- §2.1, page 127. If you have an early printing of the book where the next-to-last commutative diagram on this page is a small diagram consisting of two short exact sequences joined by vertical maps α , β , and γ , then add the hypothesis that these maps are chain maps, commuting with boundary homomorphisms. If you have a later printing with a large three-dimensional commutative diagram which includes the boundary maps as well as the maps α , β , and γ , then nothing more needs to be added. However, in the line preceding this large diagram there may be a typo in the word “sequences” in your printing of the book.
- §2.1, page 129, next-to-last paragraph. In each of the first two lines of this paragraph there is a c that should be c' .
- §2.2, page 134. The notion of degree for maps $S^n \rightarrow S^n$ is not very interesting when $n = 0$, so it may be best to exclude this case from the definition to avoid having to think about trivialities and whether H_n should be \tilde{H}_n or not.
- §2.2, page 135, last line. Add the nontriviality condition $n > 0$, to guarantee that the groups $H_n(S^n)$ in the diagram on the next page are \mathbb{Z} .
- §2.2, page 136. See the online version of the book for slightly improved phrasing of the proof of Proposition 2.30.
- §2.2, page 137, line 6. Change the word “stretching” to “shrinking.”
- §2.2, page 137. Part (c) of Lemma 2.34 has been expanded to say also that the map $H_k(X^n) \rightarrow H_k(X)$ is surjective when $k = n$. The proof on the next page has been revised to show this.
- §2.2, page 141, two lines above Example 2.36. It should be $\tilde{H}_{n-1}(X^{n-1}/X^{n-2})$, with a tilde, though it doesn’t really matter since we are in the case $n > 1$.
- §2.2, page 144, Example 2.42. The original wording here said that the restriction of $q\varphi$ to each component of $S^{k-1} - S^{k-2}$ was a homeomorphism, but it would be

clearer to specify the image of these homeomorphisms and say that $q\varphi$ restricts to a homeomorphism from each component of $S^{k-1} - S^{k-2}$ onto $\mathbb{R}P^{k-1} - \mathbb{R}P^{k-2}$.

- §2.2, page 152. In the exact sequence at the top of the page delete the final 0 and the arrow leading to it.
- §2.2, page 156, Exercise 13. The second half of part (b) should say that the only subcomplex $A \subset X$ for which the quotient map $X \rightarrow X/A$ is a homotopy equivalence is the trivial subcomplex consisting of the 0-cell alone.
- §2.2, page 158, Exercise 30, line 2. The label $1 - f_*$ on the map should be $\mathbb{1} - f_*$, with a blackboard bold 1.
- §2.2, page 158, Exercise 34. The original form of this problem was to derive the long exact sequence of homology groups for a pair (X, A) from the Mayer-Vietoris sequence. However, this is hard to do without resorting to some type of circular reasoning, so it seems best to delete this problem.
- §2.3, page 164. There is something wrong with the syntax of the long sentence beginning on line 8 of this page, the second example of a functor. The simplest correction would be to change the word “assigns” to “assigning” in line 8. Perhaps a better fix would be to break this long sentence into two sentences by putting a period at the end of line 9 and then starting a new sentence on line 10 with “This is a functor from the category \dots ”.
- §2.B, page 170. In some versions of the book there is a typo in the last line of the proof of Proposition 2B.1, where $\tilde{H}_i(S^n - h(D^k))$ was written in place of the correct group $\tilde{H}_i(S^n - h(S^k))$. (Early printings of the book used a different notation here, and the typo was only introduced when the notation was changed.)
- §2.B, page 173. In the second paragraph after Theorem 2B.5 the historical comments are in need of corrections. Frobenius’ theorem needs the hypothesis that the division algebra has an identity element, and Hurwitz only proved that the condition $|ab| = |a||b|$ implies the dimension must be 1, 2, 4, or 8. Here is a revised version of this paragraph:

The four classical examples are \mathbb{R} , \mathbb{C} , the quaternion algebra \mathbb{H} , and the octonion algebra \mathbb{O} . Frobenius proved in 1877 that \mathbb{R} , \mathbb{C} , and \mathbb{H} are the only finite-dimensional associative division algebras over \mathbb{R} with an identity element. If the product satisfies $|ab| = |a||b|$ as in the classical examples, then Hurwitz showed in 1898 that the dimension of the algebra must be 1, 2, 4, or 8, and others subsequently showed that the only examples with an identity element are the classical ones. A full discussion of all this, including some examples showing the necessity of the hypothesis of an identity element, can be found in [Ebbinghaus 1991]. As one would expect, the proofs of these results are algebraic, but if one drops the condition that $|ab| = |a||b|$ it seems that more topological proofs are required. We will show in Theorem 3.20 that a finite-dimensional division algebra over \mathbb{R} must have dimension a power of 2. The

fact that the dimension can be at most 8 is a famous theorem of [Bott & Milnor 1958] and [Kervaire 1958]. See §4.B for a few more comments on this.

■ §2.B, page 176, Exercise 3. A better hint would be to glue two copies of (D^n, D) to the two ends of $(S^{n-1} \times I, S \times I)$ to produce a k -sphere in S^n and look at a Mayer-Vietoris sequence for the complement of this k -sphere. (The hint originally given leads to problems with the point-set topology hypotheses of the Mayer-Vietoris sequence.)

■ §2.C, page 180. In the line preceding the proof of 2C.3 the S^3 should be S^4 . Also, in the line above this the reference should be to Example 4L.4 rather than to an exercise in section 4K.

■ §2.C, page 180, line -11. Typo: The formula involving τ should be just $\tau(fr) = \tau(f)$, without the star subscripts.

■ §2.C, page 180. The last sentence on this page continuing onto the next page is somewhat unnecessary since the fact that K is a subdivision of L implies that its simplices have diameter less than $\varepsilon/2$.

■ Introduction to Chapter 3, page 187. In the fourth-to-last line change “homology group” to “cohomology group”.

■ Introduction to Chapter 3, page 189, line 21. The minus sign in $\psi - \delta\varphi$ should be an equals sign.

■ §3.1, page 198, line 20. There are two missing φ 's. It should read $\varphi(\partial\sigma) = \varphi(\sigma(v_1)) - \varphi(\sigma(v_0)) = 0$.

■ §3.1, page 200. In the diagram that contains the long dashed arrow going diagonally downward there are four occurrences of the letter G . These should be deleted, along with the semicolons that precede them.

■ §3.1, page 202 line 5. Change $H^n(X, A)$ to $H^n(X, A; G)$.

■ §3.1, page 203, last line. Change the comma in $C^n(A + B, G)$ to a semi-colon.

■ §3.2, page 208. In the last sentence of the first paragraph on this page (this is the sentence referring to Theorem 3.14) it might be a good idea to add, for the sake of clarity, the phrase “assuming that the coefficient ring itself is commutative” at the end of the sentence.

■ §3.2, page 210, fifth line of Example 3.11. Insert the word “of” following “generator”.

■ §3.2, page 210, last line. $H^n(I \times Y, R)$ should be $H^n(I \times Y; R)$, with a semicolon instead of a comma.

■ §3.2, page 213, third paragraph, third line. Change $P^n - \{0\}$ to $P^n - \{p\}$.

■ §3.2, page 215. In the statement of Theorem 3.14 change “with” to “when”.

■ §3.2, page 216, first line. $C^\ell(X, R)$ should be $C^\ell(X; R)$, with a semicolon instead of a comma.

- §3.2, page 217, sixth to last line. Change “a special case of the former if $2 \neq 0$ in R ” to “a consequence of the former if R has no elements of order 2”.
- §3.2, page 218, last line of second paragraph: Change the first Y to X , so that the tensor product becomes $H^*(X; R) \otimes_R H^*(Y; R)$.
- §3.2, page 221, line 9. The strict inequality $n > i$ could be changed to $n \geq i$, although this is not important for the argument being made.
- §3.2, page 225, lines 15 and 17. Typo: Change the superscript $i + j$ on x to a subscript.
- §3.2, page 227, first sentence. The reference to the 1980 paper of Adams and Wilkerson is incorrect. In fact the proof of this fundamental result has only been completed recently in a paper of K. Andersen and J. Grodal, The Steenrod problem of realizing polynomial algebras, *Journal of Topology* 1 (2008), 747-760.
- §3.2, page 228. The algebraic problem referred to at the end of the first paragraph on this page has been solved. The answer is what one would hope: The simplicial complex C_X is uniquely determined by the cohomology ring $H^*(X; \mathbb{Z})$. In fact this is true with \mathbb{Z}_2 coefficients. A similar result holds also in the situation mentioned in the following paragraph, so a subcomplex of a product of n copies of $\mathbb{C}P^\infty$ is uniquely determined by its cohomology ring, up to permutation of the factors (and deletion of a $\mathbb{C}P^\infty$ factor if none of its positive-dimensional cells are used). The reference is Theorem 3.1 in J. Gubeladze, The isomorphism problem for commutative monoid rings, *J. Pure Appl. Alg.* 129 (1998), 35-65.
- §3.2, page 228, Example 3.24. Change Macauley to Macaulay (3 times). Also in the Index, page 540, under Cohen-Macaulay.
- §3.2, page 229, Exercise 4. The reference should be to Exercise 3 in §2.C.
- §3.2, page 229, Exercise 5. Change this to: Show the ring $H^*(\mathbb{R}P^\infty; \mathbb{Z}_{2k})$ is isomorphic to $\mathbb{Z}_{2k}[\alpha, \beta]/(2\alpha, 2\beta, \alpha^2 - k\beta)$ where $|\alpha| = 1$ and $|\beta| = 2$. [Use the coefficient map $\mathbb{Z}_{2k} \rightarrow \mathbb{Z}_2$ and the proof of Theorem 3.12.]
- §3.2, page 230. In the next to last line of Exercise 14 the exponent on α should be $2n + 1$ instead of $n + 1$.
- §3.2, page 230, Exercise 17. This can in fact be done by the same method as in Proposition 3.22, although the details are slightly more complicated. For a write-up of this, see the webpage for the book under the heading of Revisions.
- §3.3, page 234, line 7. Change “neighborhood of A ” to “neighborhood of the closure of A ”.
- §3.3, page 236. In the sixth line of the longish paragraph between Theorem 3.26 and Lemma 3.27, change the phrase “for B any open ball in M ” to “for B any open ball in M containing x .”

- §3.3, page 239, next-to-last line: Change “ $(k - \ell)$ -simplex” to “ $(k - \ell)$ -chain”. (This paragraph has been revised in later printings of the book, so this correction is no longer relevant.)
- §3.3, page 241. In the ninth-to-last line change “cycle” to “cocycle”.
- §3.3, page 242. In line 5 of the subsection *Cohomology with Compact Supports* change “chain group” to “cochain group.”
- §3.3, page 245. At the end of the first paragraph on this page it is stated that inclusion maps of open sets are proper maps, but this is not generally true. A proper map $f: X \rightarrow Y$ does induce maps $f^*: H_c^i(Y; G) \rightarrow H_c^i(X; G)$, but the proof of Poincaré duality uses induced maps of a different sort going in the opposite direction from what is usual for cohomology, maps $H_c^i(U; G) \rightarrow H_c^i(V; G)$ associated to inclusions $U \hookrightarrow V$ of open sets in the fixed manifold M .
- §3.3, page 245. In the diagram in the middle of the page the two vertical arrows are pointing in the wrong direction in the first printing of the book. This was corrected in the second printing.
- §3.3, page 247, lines 1-2. There is a missing step here. The cocycle $\delta\varphi_A$ represents $\delta[\varphi]$ as an element of $H^*(M, A+B)$ rather than $H^*(M, A \cup B)$, which is what we really want. The inclusion of $C^*(M, A \cup B)$ into $C^*(M, A + B)$ induces an isomorphism $H^*(M, A \cup B) \approx H^*(M, A + B)$, so a cocycle in $C^*(M, A \cup B)$ representing the class $[\delta\varphi_A] \in H^*(M, A + B)$ is obtained by replacing $\delta\varphi_A$ by $\delta\varphi_A + \delta\psi$ for some $\psi \in C^*(M, A + B)$. Thus we replace φ_A and φ_B by $\varphi_A + \psi$ and $\varphi_B + \psi$ and all is well.
- §3.3, page 248. In the next-to-last line of item (1) in the proof of Poincaré Duality, change “the cocycle taking” to “a cocycle φ taking”
- §3.3, page 249, line 12. Change H^{i-1} to H^{i+1} .
- §3.3, page 249. In the line above the commutative diagram two-thirds of the way down the page there are a couple missing symbols in the two Hom groups. It should read $\text{Hom}_R(C_\ell(X; R), R) \rightarrow \text{Hom}_R(C_{k+\ell}(X; R), R)$.
- §3.3, page 250. In the statement of Corollary 3.39 the condition on α should be that it generates an infinite cyclic summand of $H^k(M; \mathbb{Z})$. This is what is used in the proof, and it is stronger than the original condition of being of infinite order and not a proper multiple of another element. An example showing the difference is the group $\mathbb{Z} \times \mathbb{Z}_p$ with p prime, where the element $(p, 1)$ is not a proper multiple but it does not generate a \mathbb{Z} summand. One could also use the element $(p^n, 1)$ for any $n > 1$.
- §3.3, page 251, last line. There is a missing parenthesis following the second H^j .
- §3.3, page 252. In the fourth paragraph, just below the middle of the page, it is stated that every symmetric nonsingular bilinear form occurs as the cup product pairing in a closed simply-connected manifold with minimum homology. This is true in dimensions 4, 8, and 16 but not in other dimensions, where only the even forms are

realizable in this way. Certain other forms that are not even are realizable by manifolds with nonminimal homology (such as complex projective spaces), but it doesn't seem to be known whether all forms are realizable.

- §3.3, page 253. In the last paragraph of the proof of Proposition 3.42 it might be better to replace the subscripts i by k .
- §3.3, page 255, line 5. Omit the coefficient group Z . (It should have been a black-board bold \mathbb{Z} in any case.)
- §3.3, page 256, lines 1-2. Change the superscript 0 to a subscript, and change the two superscripts n to $n - 1$.
- §3.3, page 256, line 8. Change "Example 1.26" to "Example 1.24".
- §3.3, page 257, lines 12-14. The assertion about Čech cohomology satisfying a stronger form of excision holds for compact pairs but not in general. Perhaps the easiest correction here is simply to delete the last half of this sentence beginning with "and indeed".
- §3.3, page 258, Exercise 8, second line. Delete the second "of".
- §3.A, page 262, tenth line from the bottom. Missing prime: $b - b' = i(a)$.
- §3.B, page 268, tenth-to-last line. Change "homomorphism" to "bilinear map".
- §3.B, page 272, first line. Change "for all i " to "for all n ".
- §3.B, page 273. In the displayed equations near the bottom of the page the coefficient $(-1)^i$ in front of the last nonzero term $c \otimes \partial^2 c'$ should be deleted.
- §3.B, page 276, Corollary 3B.2 (which incidentally should have been numbered 3B.8). The isomorphism in this corollary is obtained by quoting the Künneth formula and the universal coefficient theorem, whose splittings are not natural, so the isomorphism in the corollary need not be natural as claimed. However there does exist a natural isomorphism, obtainable by applying Theorem 4.59 later in the book.
- §3.B, page 280, next-to-last line before the exercises. Change ΔT to $T\Delta$.
- §3.B, page 280, Exercise 5, lines 2 and 3. The slant products should map to the homology and cohomology of X rather than Y .
- §3.C, page 281. In the last two lines of the next-to-last paragraph, change it to read "... compact Lie groups $O(n)$, $U(n)$, and $Sp(n)$. This is explained in §3.D for $GL_n(\mathbb{R})$, and the other two cases are similar."
- §3.C, page 282, tenth line from the bottom. Change SP_{n+1} to $SP_{n+1}(X)$.
- §3.C, page 283. The summation in the displayed formula on line 14 is not sufficiently general. The formula should say

$$\Delta(\alpha) = \alpha \otimes 1 + 1 \otimes \alpha + \sum_i \alpha'_i \otimes \alpha''_i \quad \text{where } |\alpha'_i| > 0 \text{ and } |\alpha''_i| > 0$$

There are four other places in this section where a similar correction is needed. In item (2) later on the same page it should say " $\Delta(\alpha) = \alpha \otimes 1 + 1 \otimes \alpha + \sum_i \alpha'_i \otimes \alpha''_i$ " whenever $|\alpha| > 0$, where $|\alpha'_i| > 0$ and $|\alpha''_i| > 0$." Lines 3-4 on page 284 should say "so the terms α'_i and α''_i in the coproduct formula $\Delta(\alpha) = \alpha \otimes 1 + 1 \otimes \alpha + \sum_i \alpha'_i \otimes \alpha''_i$ must be zero." On page 290, item (2), it should say " $\Delta(a) = a \otimes 1 + 1 \otimes a + \sum_i a'_i \otimes a''_i$." And in item (3) on that page it should say "the lower route gives first $\Delta(a) \otimes \Delta(b) = (\sum_i a'_i \otimes a''_i) \otimes (\sum_j b'_j \otimes b''_j)$, then after applying τ and $\pi \otimes \pi$ this becomes $\sum_{i,j} (-1)^{|a''_i||b'_j|} a'_i b'_j \otimes a''_i b''_j = (\sum_i a'_i \otimes a''_i) (\sum_j b'_j \otimes b''_j)$, which is $\Delta(a)\Delta(b)$."

- §3.C, page 285, lines 7 and 8. As originally written, the definition of the coproduct in the tensor product of two Hopf algebras was not given with sufficient care. It should say: The tensor product $A \otimes B$ of Hopf algebras A and B is again a Hopf algebra, with coproduct the composition $A \otimes B \xrightarrow{\Delta \otimes \Delta} (A \otimes A) \otimes (B \otimes B) \rightarrow (A \otimes B) \otimes (A \otimes B)$ where the second map interchanging the middle two factors includes the usual sign in graded commutativity.

- §3.C, page 286, Example 3C.5, third line. Change $2i$ to ni .

- §3.C, page 286, eleventh line up from the bottom (now the ninth line up). Modify this to say "but not in $\Gamma_{\mathbb{Z}_p}[\alpha]$ when $i > 0$, since the coproduct in $\Gamma_{\mathbb{Z}_p}[\alpha]$ is given by ..."

- §3.C, page 291, Exercise 3. Assume the H-space multiplication is associative up to homotopy.

- §3.C, page 291, Exercise 9. Add the hypothesis that X is connected.

- §3.C, page 291, Exercise 10, part (c). Assume that a_n and b_n are nonzero.

- §3.C, page 293, line 16. Insert "finite-dimensional" before "CW structure".

- §3.D, page 295. In the text to the left of the figure change P^n to P^{n-1} .

- §3.D, page 297. In the last line, β^I should be β_I .

- §3.D, Proposition 3D.4. In the third line the symbol a_{2k+1} should be something different, such as a'_{2k+1} , to avoid ambiguities. The last sentence in the proposition and the last paragraph in the proof should be changed accordingly.

- §3.D, page 300, thirteenth line from the bottom. The reference should be to Theorem 3D.2 rather than Proposition 3D.2.

- §3.C, page 302, Exercise 1. Add the hypothesis that the CW structure is finite-dimensional.

- §3.C, page 304, line 11. Change the subscript p in \mathbb{Z}_p to m .

- §3.F, page 314, lines 9-10. The finite expressions $b_n \cdots b_1 b_0$ correspond just to nonnegative integers.

- §3.F, page 315, next-to-last line of first paragraph. Change H^n to h^n .

- §3.F, page 319. The proof of Proposition 3F.12 originally given was incomplete. (The gap occurred in the third paragraph on page 319 where the possibility of torsion

of order relatively prime to p was overlooked.) A corrected proof is now included in the online version of the book.

- §3.G, page 322, line 5. Change $H^k(X;F)$ to $H^k(\tilde{X};F)$.
- §3.G, pages 326-327. The list of Lie groups whose classifying spaces have polynomial \mathbb{Z}_p -cohomology rings is incomplete for the prime $p = 2$. Perhaps the best way to describe the situation would be to restrict the discussion to odd primes up until the last paragraph in this section, and then enlarge the final table for the prime 2 to include the missing examples. Among these are the following Lie groups, with corresponding polynomial generators in the indicated degrees:

G_2	4, 6, 7
$Spin(7)$	4, 6, 7, 8
$Spin(8)$	4, 6, 7, 8, 8
$Spin(9)$	4, 6, 7, 8, 16
F_4	4, 6, 7, 16, 24
$PSp(2n + 1), n \geq 1$	2, 3, 8, 12, \dots , $8n + 4$

Here $PSp(n) = Sp(n)/(\pm I)$, the quotient of $Sp(n)$ by its center. I have been told there may be other examples as well, and I will post these here when I obtain a more complete list from the experts on this subject. (Note that for $p = 2$ the term ‘degree’ means the actual cohomological dimension, whereas for odd primes it meant half the cohomological dimension.)

- §3.H, page 332, line -9. Change “Bockstein” to “change-of-coefficient”.
- §3.H, page 333, line 13. Change “bundles of groups” to “bundles of abelian groups”.
- §3.H, page 334, line 2. Missing parenthesis in $C^n(X;E)$.
- §3.H, page 334, end of line 5. Typo: Change $\gamma \in G$ to $\gamma \in \pi$.
- §3.H, page 334, line following Proposition 3H.5. Repeated “the” — delete one.
- §3.H, page 335. In the statement of Theorem 3H.6, Poincaré duality with local coefficients, change the second (or alternatively, the third) occurrence of M_R to R , just ordinary coefficients in R rather than local coefficients. For more details see the separate correction page.
- §3.H, page 336, Exercise 5. The assertion that $H^1(X; \mathbb{Z}[\pi_1 X])$ is an infinite direct sum of copies of \mathbb{Z} holds only when $\pi_1(X)$ is free on two or more generators. When $\pi_1(X)$ is infinite cyclic the cohomology group is just a single \mathbb{Z} .
- §3.H, page 336, Exercise 6. In the last part of the question add the assumption that X is finite-dimensional.
- §4.1, page 339, second line of last paragraph. The reference should be to §4.B instead of §4.C.
- §4.1, page 345, line 2. Change (X, B, x_0) to (X, A, x_0) .

- §4.1, page 348. The first paragraph of the subsection on Cellular Approximation has been revised to eliminate a reference to an earlier proof of Proposition 1.14 that has been replaced by a different proof in later versions of the book.
- §4.1, page 349, line 10. Delete the word “to” preceding “try”.
- §4.1, page 349, thirteen lines from the bottom. It should perhaps be mentioned that the deformation of f on e^k to make $f(e^k)$ miss the point p will not make $f(e^k)$ intersect any more cells than it intersected before.
- §4.1, pages 350-351. The statement and proof of Lemma 4.10 have been revised a couple times. The statement was revised again in October 2012 to say explicitly that the homotopy takes $f^{-1}(e^k)$ to e^k at all times. The proof gives this additional property, and this property is needed when Lemma 4.10 is used in the proof of Theorem 4.23, Case 1, later in the chapter.
- §4.1, page 354, eighth line up from the bottom. Delete one of the duplicated words “be”.
- §4.1, page 358, Exercise 9, first line. To avoid an abuse of notation, replace $\pi_0(A, x_0)$ by $i^*(\pi_0(A, x_0))$.
- §4.1, page 359, Exercise 22. Add the word “weakly” before “homotopy equivalent”.
- §4.2, page 361, line 18. Repeated “the” — delete one of them.
- §4.2, page 361. The latter part of the paragraph preceding the figure has been reworded for clarity. See the online version of the book. (Another slight rewording: January 2010)
- §4.2, page 362, line 19. Replace I^{n-1} by I^{i-1} .
- §4.2, page 370. The large diagram on this page will only commute up to sign unless the generators α are chosen carefully. Commuting up to sign is good enough for most purposes, so this isn’t really a big issue. It might be a good exercise to see how to choose generators to make the diagram commute exactly.
- §4.2, page 371, Twelfth line from the bottom. Wrong font for the symbol X near the beginning of this line. (Should be italic.)
- §4.2, page 371, next-to-last line. Change (W, X^1) to (W, X) .
- §4.2, page 374. Delete the direct sum symbol \oplus at the end of the displayed exact sequence in the sixth line.
- §4.2, page 376. In the proof of injectivity of p_* there is an implicit permutation of the last two coordinates of $I^n \times I$ when the relative homotopy lifting property is applied.
- §4.2, page 380. At the end of Example 4.50 replace $K(\mathbb{Z}, 3)$ by $K(\mathbb{Z}, 4)$.
- §4.2, page 385. In versions of the book before 2016 the chart showing the 2-primary parts of the stable homotopy groups of spheres had a couple of errors in

the range above dimension 50. The original calculations in this range were done by Kochman and Mahowald in the 1990's. When these groups were recalculated by Dan Isaksen by different methods in a 2014 arXiv preprint called "Stable stems", a few discrepancies were found. Isaksen's calculations have been checked by other experts, so there is a high probability that they are correct. A corrected version of the chart now appears in the online version of the book. A few changes were also made in the accompanying text in pages 385-388.

- §4.2, page 389, Exercise 11. There are counterexamples to the second half of this problem as originally stated, which involved an analog of the first half with π_2 replaced by π'_2 . The current online version of this exercise includes such a counterexample.
- §4.2, page 390, Exercise 15. The Poincaré conjecture has been proved.
- §4.2, page 391, line 5. $H_n(X)$ should be $H_{n+1}(X)$.
- §4.2, page 391, Exercise 25. The CW complex X is assumed to be connected, as is implicit in the notation $\pi_n(X)$ without a basepoint.
- §4.2, page 391, Exercise 27. This exercise can be done directly from the definition of relative homotopy groups, so it really belongs in §4.1.
- §4.3, page 394, third paragraph. The hypothesis that X be connected is unnecessary. Also, a comment could be added at the end of the paragraph that $H^0(X; G) = [X, K(G, 0)]$ and $\tilde{H}^0(X; G) = \langle X, K(G, 0) \rangle$.
- §4.3, page 398, line 3. Change SX to SA .
- §4.3, page 399, third paragraph. Change L to K' , twice.
- §4.3, page 399, middle. The label (4) on the displayed exact sequence can safely be omitted.
- §4.3, page 400, line 6. Replace $h^n(\text{point})$ by $h_n(\text{point})$. (This typo crept in when I modified this sentence some time after the first printing, so it doesn't occur in the first printing.)
- §4.3, page 403. Sixteen lines from the bottom, change z to y twice in this line, for notational consistency with the use of y earlier in the paragraph.
- §4.3, page 409, next-to-last line of next-to-last paragraph. Switch y and η , so that it reads "composing the inverse path of $p\eta$ with y ."
- §4.3, middle of page 412. In the definition of the k -invariant the coefficient group should be $\pi_{n+1}(X)$ instead of $\pi_{n+1}(K)$. (For consistency, the parentheses surrounding this X can be deleted.) Another correction: In the line preceding this, change π_{n+1} to $\pi_{n+1}X$.
- §4.3, page 417, last line. The reference should be to Lemma 4.7 rather than to an exercise in §4.1.

- §4.3, page 418. In the paragraph containing the diagram it should be stated, for the sake of clarity, that F is the fiber of the fibration $X \rightarrow Y$.
- §4.3, page 419, Exercise 6. It should have been explained how the cross product is defined since we are using coefficients in G rather than a ring. However, instead of using cross products it would be better just to use Exercise 4 to construct the H-space structure and prove the stated properties. The problem could also be expanded to include showing that the H-space structure has a homotopy-inverse.
- §4.3, page 419, Exercise 8. Typo in the second line: ps should be πs .
- §4.3, page 420, Exercise 13. Small typo: It should begin "Given a map".
- §4.A, page 422, second and sixth lines from the bottom. It should be $\mathbb{Z}[\pi_1(X)]$ instead of $\mathbb{Z}[\pi_n(X)]$.
- §4.A, page 425, eleventh and tenth lines from the bottom. Change "octagon" to "octahedron".
- §4.B, page 428, line 6. Typo: Replace Adam' by Adams'.
- §4.C, page 429. In the line preceding the diagram, change Z_{n+1} to Z^{n+1} .
- §4.C, page 430, third line of Example 4C.2. Insert the word "and" before $H_{n+1}(X)$.
- §4.D page 438, line 13. The tensor product should be over the ring R , so add a subscript R to the tensor product symbol.
- §4.D, page 445, eleventh line from the bottom. Change "corollary" to "proposition".
- §4.D, page 447, exercise 8. In the second line replace $\Sigma^n B$ by $\Sigma^n(B_+)$ where B_+ is the union of B with a disjoint basepoint. Also, in the last part of the problem the cohomology isomorphism should be $\tilde{H}^i(B; R) \approx \tilde{H}^{n+i}(\Sigma^n B; R)$ with both groups reduced.
- §4.E, page 448, fourth line after Theorem 4E.1. Change the R to G as the coefficient group.
- §4.E, page 448. In the diagram near the bottom of the page all the A 's should be in the same italic font.
- §4.E, page 449, tenth line from the bottom. Change the word "two" to "a few" (since there are now three comments — see the next correction).
- §4.E, page 450. There is a gap in the proof of Lemma 4E.4 (fifth sentence) that can be filled by adding an item (3) after the first paragraph on page 450:

(3) If h satisfies axioms (i) and (iii) then $h(\Sigma Y)$ is a group and $T_u : \langle \Sigma Y, K \rangle \rightarrow h(\Sigma Y)$ is a homomorphism for all suspensions ΣY and all pairs (K, u) . The group structure comes from the map $c : \Sigma Y \rightarrow \Sigma Y \vee \Sigma Y$ collapsing an equatorial copy of Y in ΣY to a point, which induces an addition operation $h(\Sigma Y) \times h(\Sigma Y) \approx h(\Sigma Y \vee \Sigma Y) \rightarrow h(\Sigma Y)$. Associativity follows from the fact that the two compositions $\Sigma Y \rightarrow \Sigma Y \vee \Sigma Y \rightarrow \Sigma Y \vee \Sigma Y$, where the first map is c and the second is either $c \vee \mathbb{1}$ or $\mathbb{1} \vee c$, are

homotopic. To show that the distinguished element $0 \in h(\Sigma Y)$ is an identity for the group operation, consider the composition $\Sigma Y \rightarrow \Sigma Y \vee \Sigma Y \rightarrow \Sigma Y$ where the first map is c and the second map collapses one of the two summands to the basepoint so it sends an element $x \in h(\Sigma Y)$ to $(x, 0)$ or $(0, x)$ in $h(\Sigma Y) \times h(\Sigma Y)$, hence the composition of the two maps sends x to $x + 0$ or $0 + x$. Since the composition $\Sigma Y \rightarrow \Sigma Y$ is homotopic to the identity, this says $x + 0 = x = 0 + x$. For inverses, let $x \mapsto -x$ be the map on $h(\Sigma Y)$ induced by the map $r: \Sigma Y \rightarrow \Sigma Y$ reversing the ends of the I factor of ΣY . Consider the composition $\Sigma Y \rightarrow \Sigma Y \vee \Sigma Y \rightarrow \Sigma Y \vee \Sigma Y \rightarrow \Sigma Y$ where the first map is c , the second is $\mathbb{1} \vee r$ or $r \vee \mathbb{1}$ and the third map identifies the two copies of ΣY . These maps send $x \in h(\Sigma Y)$ to (x, x) , then to $(x, -x)$ or $(-x, x)$, then to $x + (-x)$ or $(-x) + x$. Since the composition $\Sigma Y \rightarrow \Sigma Y$ is homotopic to the constant map, this says $-x$ is an additive inverse to x . Thus we have a group structure on $h(\Sigma Y)$. It remains to see that $T_u: \langle \Sigma Y, K \rangle \rightarrow h(\Sigma Y)$ is a homomorphism. The sum of maps $f, g: \Sigma Y \rightarrow K$ is given by the composition $\Sigma Y \rightarrow \Sigma Y \vee \Sigma Y \rightarrow K$ of c with $f \vee g$. This composition takes $u \in h(K)$ to $(f + g)^*(u)$, while $f \vee g$ takes u to $(f^*(u), g^*(u))$ and c takes this to $f^*(u) + g^*(u)$, so $(f + g)^*(u) = f^*(u) + g^*(u)$ which says that $T_u(f + g) = T_u(f) + T_u(g)$, and so T_u is a homomorphism.

■ §4.E, page 450. Once the new item (3) has been added to this page, the sentence in the paragraph before Lemma 4E.3 beginning “Note that having a trivial kernel” should be deleted.

■ §4.E, page 450. In the third-to-last line T_n should be T_{u_n} , and again in the next line as well.

■ §4.F, page 454, two lines above Proposition 4F.2. Typo: change $\varinjlim \pi_{i+n}^s(K_n)$ to $\varinjlim \pi_{i+n}^s(X \wedge K_n)$.

■ §4.F, page 454. In the last paragraph it is stated that one can associate a cohomology theory to any spectrum by setting $h^i(X) = \varinjlim \langle \Sigma^n X, K_{n+i} \rangle$. Unfortunately the wedge axiom fails with this definition. For finite wedges there is no problem, so one does get a cohomology theory for finite CW complexes. A way to avoid this problem is to associate an Ω -spectrum to a given spectrum in the way explained on the next page, then take the cohomology theory associated to this Ω -spectrum.

■ §4.G, page 456, thirteenth line up from the bottom. Change $X_1 \hookrightarrow X_1$ to $X_0 \hookrightarrow X_1$.

■ §4.H, page 463, line 4. Delete the extra period at the end of the paragraph.

■ §4.H, page 463, line 5. Typo: let us consider.

■ §4.H, page 464, line 14. The superscript on D should be n rather than m .

■ §4.I, page 467, line 9. Change $S^1 \hookrightarrow S^1$ to $S^1 \hookrightarrow S^1 \vee S^1$.

■ §4.I, page 468. Exactly halfway down the page the term $J_n(X)$ should be $\Sigma J_n(X)$.

■ §4.I, page 470, Exercise 2. In the first line there are three missing Σ 's. It should say $\Sigma K(\mathbb{Z}_m \times \mathbb{Z}_n, 1) \simeq \Sigma K(\mathbb{Z}_m, 1) \vee \Sigma K(\mathbb{Z}_n, 1)$. Also, in the last line the reference should be to Proposition 4I.3 instead of 4E.3.

- §4.I, page 470, Exercise 3. The lens space should be assumed to be of high dimension.
- §4.J, page 473. At the end of the paragraph containing the commutative diagram, add “by Example 4A.3”.
- §4.K, page 480. In the statement of part (b) of Lemma 4K.3, instead of assuming that B has the weak or direct limit topology, assume that each compact set in B is contained in some B_n . (This is to avoid point-set topology issues.) In the electronic version of the book the proof of this lemma has also been revised slightly, clarifying basepoint issues in parts (a) and (b) and simplifying the proof in (c).
- §4.K, page 482. In Example 4K.5 it is the unreduced suspension rather than the reduced suspension that is being used, so to be consistent with the notation elsewhere in the book, each of the five occurrences of the symbol Σ in this example should be replaced by S .
- §4.L, page 488, first sentence of the proof of Proposition 4L.1. The identification $H^m(X;G) = \langle X, K(G, m) \rangle$ is valid only for $m > 0$. For $m = 0$ one has $H^0(X;G) = [X, K(G, 0)]$.
- §4.L, page 488, next-to-last line. It would be better to say that there are no non-trivial cohomology operations that decrease dimension.
- §4.L, page 491, seventh line from the bottom. The exponent $n + 4i$ should be $n + 2i$. The same correction should be made again on the second line of the next page.
- §4.L, page 493. Replace the two sentences immediately preceding Example 4L.5 by the following: “In particular, d is not equal to -1 . The Lefschetz number $\lambda(f) = 1 + d + \cdots + d^n = (d^{n+1} - 1)/(d - 1)$ is therefore nonzero since the only integer roots of unity are ± 1 . The Lefschetz fixed point theorem then gives the result.”
- §4.L. Starting on page 496 and continuing for the rest of this section the name Adem is mistakenly written with an accent, as Adém. (In fact the name is pronounced with the accent on the first syllable.)
- §4.L, page 500, sixth-to-last line. Change 4K.1 to 4L.1.
- §4.L, page 501, line 15. In the displayed formula the signs on the two occurrences of the index j in the exponents should be reversed, so the formula reads

$$\sum_j \binom{k}{j} Sq^{2n-k+j-1} Sq^{n-j} = 0$$

The same correction needs to be made in the analogous formula involving Steenrod powers near the end of this paragraph.

- §4.L, page 503, line 5. Typo: The word “definition” should be “definitions”.
- §4.L, page 503, eighth and ninth lines from the bottom. Change $(p - 1)n$ to $(p - 1)n^2$ twice.

- §4.L, pages 504-505, second and third paragraphs of the proof of Theorem 4.12. There is a mistake here since λ is in fact not additive. Fortunately there is a simple way to deduce additivity of Sq^i from the other axioms, and this argument is now given in the online version of the book. (Correcting this proof has produced changes in the page breaks for pages 502-513.)
- §4.L, page 509. There are sign problems in the proof of Lemma 4L.14. For a corrected version of the argument see the online version of the book.
- Appendix, page 521. The statement of condition (i) in Proposition A.2 has been revised for clarity, to avoid an implicit dependence on condition (ii). The paragraph following the proposition has been revised accordingly.
- Appendix, page 528, ninth line from the bottom. At the beginning of the line change X to X_{2i-1} .
- Appendix, page 530, Proposition A.14. The definition of local compactness we are using here is that each neighborhood of each point contains a compact neighborhood of the point. This follows the general pattern described on page 62, but it is stronger than the more common definition which is that each point has at least one compact neighborhood. For Hausdorff spaces the two definitions agree.
- Appendix, page 532, last line of the proof of Proposition A.16. Typo: Change $(X^Y)^Z$ to X^Y .
- Appendix, page 532, the added section on the Homotopy Extension Property. In the first line of this added section change the reference to Chapter 1 to Chapter 0, and in the second line the word “certain” is misspelled.
- Appendix, page 533, fourth-to-last line of the proof of Proposition A.18 (this proposition was only added to the Appendix in 2009). Add a bar over the symbol A . Also the rest of this sentence should say that r_1 is a continuous map to X and $X \cap O$ is open in X .
- Index, page 541. In the entry for the Hurewicz theorem the first two page numbers should be 366 and 371.