Addendum to CERF THEORY FOR GRAPHS

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The proof of Proposition 6.2 neglected to mention a key point, so we clarify this by adding a few sentences to the proof. Proposition 6.2 asserts that the subcomplex C_n of the spine $S\mathbb{A}_n$ of $\mathbb{A}_n$ consisting of basepointed graphs for which the basepoint is a cut vertex is $(n-3)$-connected.

Proof of Proposition 6.2. Consider a map $S^i \to C_n$ where $i \leq n-3$. Since $S\mathbb{A}_n$ is contractible, this extends to a map $D^{i+1} \to S\mathbb{A}_n$. By the Degree Theorem, this extension can be deformed into $S\mathbb{A}_{n,n-2}$. The proof of the Degree Theorem shows that over S^i the basepoint remains a cut vertex during the deformation, so the deformation is a homotopy of maps of pairs $(D^{i+1}, S^i) \to (S\mathbb{A}_n, C_n)$. Alternatively, this can be deduced from the fact that the degree decreases monotonically during the deformation, as follows. For any path in $S\mathbb{A}_n$ the topological type of a graph only changes when one passes from a simplex to one of its faces, collapsing a subtree to a point, or the reverse of this process, expanding a vertex to a subtree. Collapsing a subtree cannot change a cut vertex to a non-cut vertex. Expanding a non-basepoint vertex does not change the basepoint cut vertex to a non-cut vertex. Expanding the basepoint cut vertex can change it to a non-cut vertex, but this will decrease the valence of the basepoint and thus increase the degree.

By part (iii) of Lemma 5.2 the base vertex of any pointed graph of rank n and degree at most $n-2$ is a cut vertex, so that $S\mathbb{A}_{n,n-2} \subset C_n$, proving the proposition. $\square$