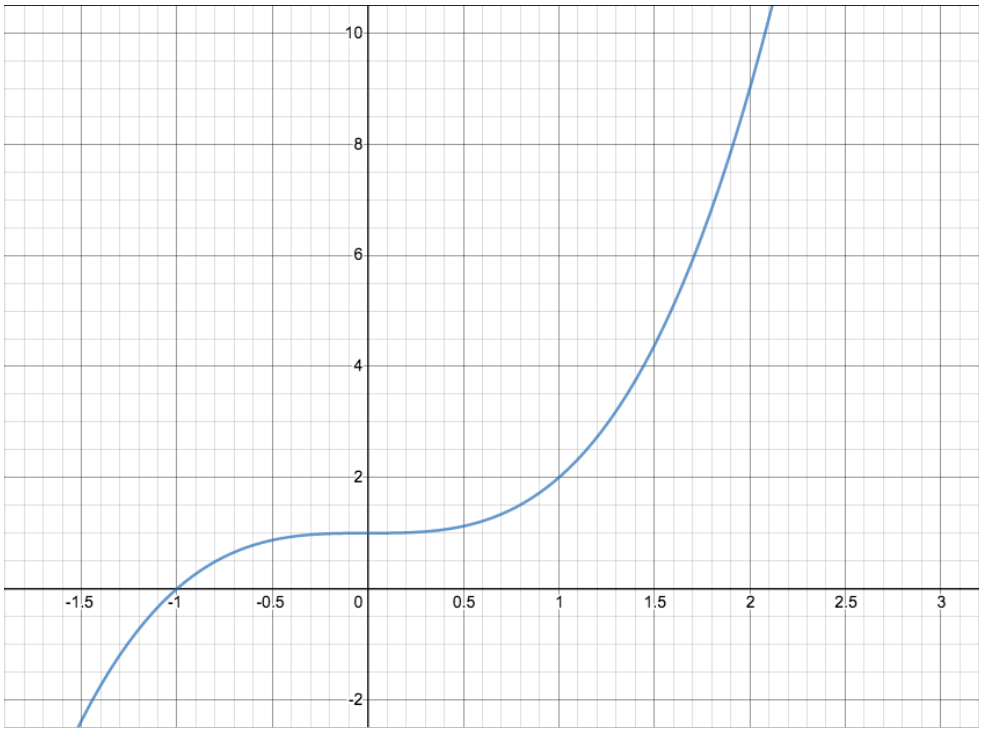
1 Rates of change and tangent lines

The height in meters with respect to time in minutes of one of the ospreys under observation by the Cornell Lab of Ornithology follows the function $h(t) = t^3 + 1$, which is graphed below.



Let's study the movement of the osprey starting at $t = 1$ minute.

1. Complete the following table. For the first three rows of the table, draw the corresponding secant lines through the endpoints of the intervals. You can use a calculator for the computations.

Interval $[1, t]$	Change in h: $h(t) - h(1)$	Slope of the secant line of h on $[1, t]$
$[1, 2]$		
$[1, 1.5]$		
$[1, 1.1]$		
$[1, 1.01]$		
$[1, 1.001]$		

2. What do you notice about these values? Do they seem to stabilise and approach a particular number?

3. Consider the following function:

$$v(t) = \frac{h(t) - h(1)}{t - 1}$$

   (a) What aspect of the movement of the osprey is $v(t)$ measuring?

   (b) What happens to $v(t)$ at $t = 1$?

   (c) Do the values of $v(t)$ approach a particular number as t approaches 1?

   (d) How could we measure the instantaneous velocity of the osprey at $t = 1$?

2 Limits at infinity

Darnell and Natalie then discuss the limit at infinity of the function $g(x) = x\sin(1/x)$:
DARNELL: The limit $\lim_{x\to\infty} x = \infty$ *so by the multiplicative limit law, we also have* $\lim_{x\to\infty} g(x) = \infty$.
NATALIE: But on the graph of the function, it looks like the curve flattens out. Since $\lim_{x\to\infty} 1/x = 0$, *the multiplicative law says the limit of* g *should be 0.*
DARNELL: If we set $y = 1/x$, *we get* $\lim_{x\to\infty} g(x) = \lim_{y\to\infty} \frac{\sin(y)}{y} = 0$, *so we do get 0 after all.*

Explain any mistakes or incomplete arguments Natalie and Darnell have made. If a statement requires justification, provide it. What is the limit $\lim_{x\to\infty} g(x)$?

3 Squeeze theorem and existence of limits

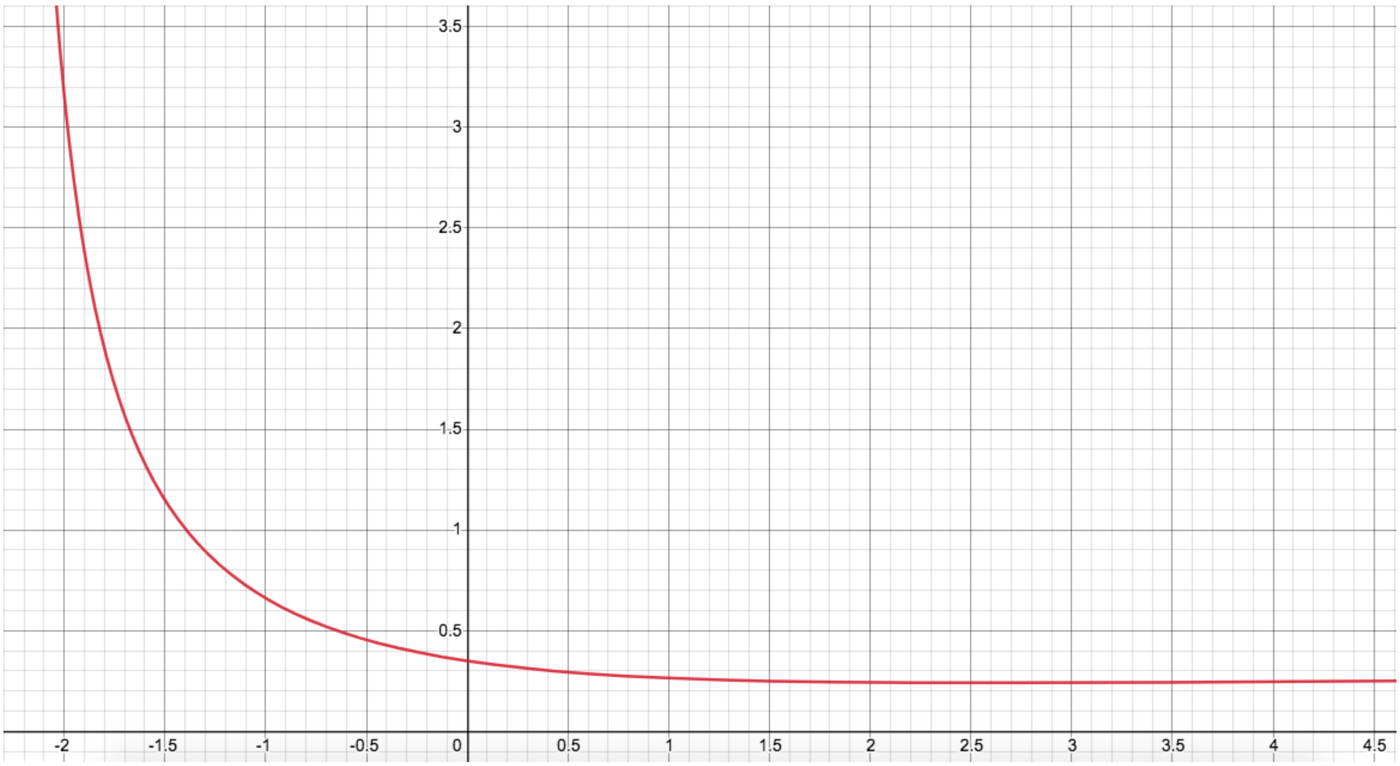
True or False? For the following questions, determine whether the statements are true or false. Justify your answers.

1. If we replace the inequalities in the statement of the squeeze theorem by strict inequalities, the theorem would no longer be true.

2. If $\lim_{x\to a} g(x) = 0$ then for any function $f(x)$, $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist.

4 Areas under curves

Consider the following graph. Our aim is to approximate the area under the curve as best we can.



1. List **as many ways as you can** to estimate the area under a curve (note that this does not include guessing what the graph might be and using a computer program!). Make sure that these methods work for a general graph, not just this one!

2. Together with your teammates, use any method you like to estimate the area under the curve on the interval $[-2, 4]$. Try to make the estimate as good as possible while still being time-efficient. Determine whether you are under-estimating or over-estimating the area. The team with the closest approximation wins!