Corrections to the First Printing: Inside Cover, Chapter 0 and Chapter 1

Corrections to Inside Cover

Notation particular to this book: the definition of $\overline{\mathbb{R}}$ should read "$\mathbb{R} \cup \{\pm\infty\}$," not "$\mathbb{R}$ excluding $\pm\infty$."

Corrections to Chapter 0

p.2: In the list of Greek letters, φ, φ should be φ, ϕ; both are versions of phi.

p. 9, Equation 0.4.3, sup and inf are undefined; see Definitions 1.6.4 and 1.6.6.

p. 9: In Definition 0.4.4, l is an integer: " ... there exists an integer l ... ."

p. 9, Definition 0.4.5: "for each $i = 1, \dots, n$," not "for each $i = 0, \dots, n$."

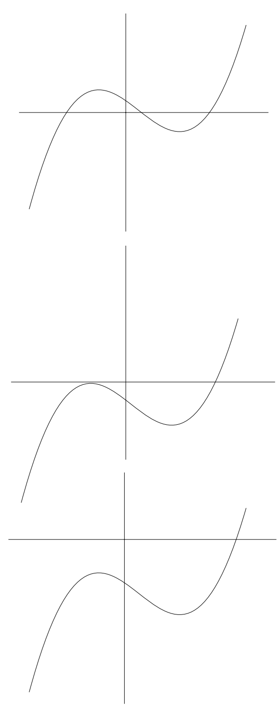
p. 9: In Proposition 0.4.6, it should read "and which satisfies the continuity condition that for all...," not "and which satisfies that the continuity condition for all..."

p 11 Since Theorem 0.4.10 is an "if and only if" statement, the proof should deal with both directions; thus it should start, "If a sequence a_n converges, it is clearly bounded. If it is bounded, it has a least upper bound A."

pp. 13–14: Russell wrote to Gottlob Frege, not to Cantor.

p. 17 Part (3) of the proof of Proposition 0.6.5: "do not differ by an integer multiple of 2π," not "by a multiple of 2π.

p.20 Figure 0.6.3 does not correspond to the caption. The figure should be



p. 21 Exercise 0.4.3: we think this will be clearer if the last sentence reads, " Notice that for A and S, the l of Definition 0.4.4 does not depend on N, but that for M, l does depend on N."

p. 24 Exercise 0.6.6, part (c): This should be "Show that there exists an angle θ such that $\cos(3\theta) = q_1$," not " ... such that $3\theta = q_1$."

Corrections to Chapter One

p. 33, line 1: "A non-empty subset $V \subset \mathbb{R}^n$" not "A non-empty subset $V \in \mathbb{R}^n$."

p. 39 Two lines before Proposition 1.2.8, "Is $(AB)C$ the same as $(AB)C$?" should be "Is $(AB)C$ the same as $A(BC)$?'.

p. 46 The top of the page might be clearer with a few changes:

First notice that $B_1 = A^1 = A$: the entry $A_{i,j}$ of the matrix A is exactly the number of walks of length 1 from v_i to v_j.

Next, suppose it is true for n, and let us see that it is true for $n+1$. A walk of length $n+1$ from V_i to V_j must be at some vertex V_k at time n. The number of such walks is the sum, over all such V_k, of the number of ways of getting from V_i to V_k in n steps, times the number of ways of getting from V_k to V_j in one step (which will be 1 if V_k is next to V_j, and 0 otherwise).

p. 48, remark: We are informed that some people who use the word "range" (rather than "image") to denote the elements of the target space that are actually reached use the word "codomain" for the entire space of arrival.

p. 48: In the footnote, the next-to-last sentence might be clearer as follows: "At the time it was viewed as pathological but it turns out to be important for understanding Newton's method for complex cubic polynomials" (i.e., "for complex cubic polynomials" rather than "in the complex").

p. 64 Second margin note: the mention of "Minkowski norm" appears to be incorrect. What we call the length of a matrix is called the Frobenius norm, the Schur norm and Hilbert-Schmidt norm.

p. 66, proof of Proposition 1.4.14: we should have said that θ is the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$.

p. 77 Add commas two lines before Equation 1.5.10, and in Equation 1.5.11: $M_1, \dots, M_n$.

p. 82 The line immediately before Equation 1.5.22 should read "If $|\mathbf{x} - \mathbf{x}_0| < \delta$, then $|\mathbf{x} - \mathbf{x}_0| < \delta_i$, so that $|f_i(\mathbf{x}) - a_i| < \epsilon$, so that

p. 86 First sentence: replace "There is a reformulation in terms of epsilons and deltas" by

"The following criterion shows that it is enough to consider $\mathbf{f}$ on sequences converging to $\mathbf{x}_0$."

Replace Proposition 1.5.26 by

Proposition 1.5.26 (Criterion for continuity). *The map $\mathbf{f} : X \to \mathbb{R}^m$ is continuous at $\mathbf{x}_0$ if and only if for every sequence $\mathbf{x}_i \in X$ converging to $\mathbf{x}_0$,*

$$\lim_{i \to \infty} \mathbf{f}(\mathbf{x}_i) = \mathbf{f}(\mathbf{x}_0).$$

p. 86 Theorem 1.5.27 part (e) should read:

If h is continuous at $\mathbf{x}_0$, with $h(\mathbf{x}_0) = 0$, and $\mathbf{f}$ is bounded is a neighborhood of $\mathbf{x}_0$, then $h\mathbf{f}$ is continuous at $\mathbf{x}_0$ (even if $\mathbf{f}$ is not defined at $\mathbf{x}_0$).

p. 90 Three lines above Equation 1.6.1, "a second box $B_1 \subset B_0$ of side 1/10," not "a second box $B_1 \in B_0$ of side 1/10."

p. 93: In Definition 1.6.6, third line, "such that $f(a) \geq y$ for all $a \in C$," not "such that $f(a) \geq x$ for all $a \in C$."

p. 94: the margin note should read

> Notice that the theorem does not require that the derivative be continuous. But it does require that the derivative take on a value at every point (that's what being differentiable means). Thus (surprise!) a car cannot jump from going 59 mph to going 61 mph, without ever passing through 60 mph. We will see in Example 1.9.4 a differentiable function with a discontinuous derivative. (See also Exercise 0.4.7.) This kind of oscillating discontinuity is the only kind of discontinuity a derivative can have; this is more or less what the mean value theorem says. More precisely, the derivative of a differentiable function satisfies the intermediate value property.

p. 98 , Equation 1.6.22 is wrong: it should be

$$\big(r(\cos\theta + i\sin\,\theta)\big)^k = r^k(\cos k\theta + i\,\sin k\theta).$$

(This equation was stated correctly in Corollary 0.6.4)

p. 99: Three lines before the end of the page: fundamental theorem of algebra, not fundamental theorem of calculus.

p. 101, right after Definition 1.7.1, "about open sets $U \subset \mathbb{R}$" not "about open sets $U \in \mathbb{R}$."

p. 106 There should be parentheses in the right-hand sides of Equations 1.7.18 and 1.7.19:

$$\begin{aligned}\lim_{h\to 0}\frac{1}{h}\Big(\big(f(a+h)-f(a)\big)-[f'(a)]h\Big) &= \lim_{h\to 0}\Big(\frac{f(a+h)-f(a)}{h} - \frac{f'(a)h}{h}\Big)\\ &= f'(a)-f'(a)=0. \qquad 1.7.18\end{aligned}$$

and

$$0 = \lim_{h\to 0}\frac{1}{h}\Big(\big(f(a+h)-f(a)\big)-mh\Big) = \lim_{h\to 0}\left(\frac{f(a+h)-f(a)}{h} - \frac{mh}{h}\right) = f'(a)-m, \qquad 1.7.19$$

p. 107 Definition 1.7.8 should specify the dimensions of $\mathbf{f}$: "The Jacobian matrix of a function $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$ is the $m \times n$ matrix ... ."

p. 111 Margin note: "...gives $f\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$, and we are asking..." not "gives $f\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$."

p. 115: The following margin note has been added:

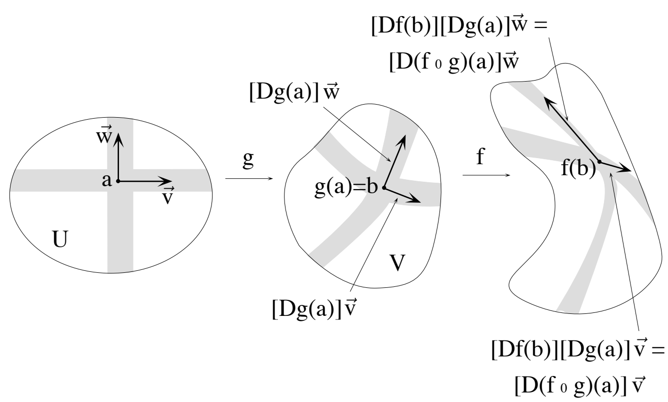
In Theorem 1.8.1, we considered writing $\mathbf{f}$ and $\mathbf{g}$ in as $\vec{\mathbf{f}}$ and $\vec{\mathbf{g}}$, since some of the computations only make sense for vectors: for example, the dot product $(\mathbf{f}\cdot\mathbf{g})(x)$. We did not do so partly to avoid heavy notation, but also because these rules often are applied in computations where one is not thinking in terms of points or vectors. In practice, you can go ahead and compute without worrying about the distinction.

p. 116 In Equation 1.8.4, it might be clearer to write $[\mathbf{D}f(\mathbf{a})]\vec{\mathbf{v}}$ rather than $f'(\mathbf{a})\vec{\mathbf{v}}$. In the last sentence of the first margin note, $[\mathbf{Dg}(\mathbf{a})]\vec{\mathbf{v}}$ should be replaced by $[\mathbf{D}f(\mathbf{a})]\vec{\mathbf{v}}$

p. 117, In Equation 1.8.8, $\frac{1}{\vec{\mathbf{h}}}$ should be $\frac{1}{|\vec{\mathbf{h}}|}$ in three places (one on each line).

p. 117 In part (4) of the proof, f and g should be $\mathbf{f}$ and $\mathbf{g}$.

p. 118 There are errors in the labels of figure 1.8.1 and in the caption. It should be



Caption: The function $\mathbf{g}$ maps a point $\mathbf{a}\in U$ to a point $\mathbf{g}(\mathbf{a})\in V$. The function $\mathbf{f}$ maps the point $\mathbf{g}(\mathbf{a})=\mathbf{b}$ to the point $\mathbf{f}(\mathbf{b})$. The derivative of $\mathbf{g}$ maps the vector $\vec{\mathbf{v}}$ to $[\mathbf{Dg}(\mathbf{a})](\vec{\mathbf{v}})$. The derivative of $\mathbf{f}\circ\mathbf{g}$ maps $\vec{\mathbf{v}}$ to $[\mathbf{Df}(\mathbf{b})][\mathbf{Dg}(\mathbf{a})]\vec{\mathbf{v}}$, i.e., to $[\mathbf{Df}(\mathbf{g}(\mathbf{a}))][\mathbf{Dg}(\mathbf{a})]\vec{\mathbf{v}}$.

p. 121 Equation 1.9.5 is missing an "as $s\to 0$"; it should be

$$g'(t_0)=\lim_{s\to 0}\frac{g(t_0+s)-g(t_0)}{s}=\lim_{s\to 0}\frac{f\big(\mathbf{c}+s(\mathbf{b}-\mathbf{a})\big)-f(\mathbf{c})}{s}=[\mathbf{D}f(\mathbf{c})](\mathbf{b}-\mathbf{a}).$$

p. 128: The margin note next to Exercise 1.2.2 may be confusing, as the matrices are the same as in Exercise 1.2.2, part (a), but in opposite order. It should read

remember to use the format:

$$\begin{array}{cc} & \begin{bmatrix} 7 & 8 \\ 9 & 0 \\ 1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} & \begin{bmatrix} .. & .. \\ .. & .. \end{bmatrix} \end{array}$$

p. 129 Exercise 1.2.11, part (b) should be with the exercises to Section 1.4.

p. 130 Exercise 1.2.15 (e) should read

Show that you can color the vertices of a connected graph (one on which you can walk from any vertex to any other) in two colors, such that no two adjacent vertices have the same color, if and only if for all sufficiently high powers n of the adjacency matrix, those entries that are 0 for A^n are nonzero for A^{n+1}, and those that are nonzero for A^n are zero for A^{n+1}.

p. 131 Some students found the wording of Exercise 1.2.22 confusing. It should read

What 2×2 matrices A satisfy

$$\text{(a) } A^2 = 0, \qquad \text{(b) } A^2 = I, \qquad \text{(c) } A^2 = -I?$$

p. 132 Exercise 1.3.11, each homework counts for 2.5 percent of the final grade, not 1.5 percent, and we should have asked, "What matrix operation should one perform to assign to each student his or her final grade?"

p. 133 Exercises 1.3.18 and 1.3.19: to be consistent with notation elsewhere in the text, $\Re$ should be Re.

p.134, Exercise 1.4.8 is the same as 1.4.4(c)

p. 135 Exercise 1.4.13 (a) The last point should be $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$, not $\begin{pmatrix} 6 \\ 7 \end{pmatrix}$

p. 136, Exercise 1.5.4, the first part of part (c) should have e^A, not a^A:

Do you think that $e^{A+B} = e^A e^B$ for all A and B?

Part (c 2) should read

Do you think that $e^{2A} = \left(e^A\right)^2$ for all A?

p. 137 Exercise 1.5.8: Here and elsewhere, log means natural logarithm (log base e). Because this appears to lead to some confusion, in future printings it will be replaced by ln, and log will only mean the "common logarithm," log base 10.

p. 137 Exercise 1.5.9 should read

What subset of $\mathbb{R}$ is the natural domain of the function

$$(1+x)^{1/x} \quad \text{of Example 1.5.18?}$$

p. 137 Exercise 1.5.10 shouldn't be there (it requires material no longer in the book).

p. 138 Exercise 1.5.12: to be consistent with notation elsewhere and in the solutions, in the second printing ϕ will be changed to φ.

p. 138, Exercise 1.5.15 (b) should be:

For the two functions below, defined on $\mathbb{R}^2 - \left\{\begin{pmatrix} 0 \\ 0 \end{pmatrix}\right\}$, either show that the limit exists at $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and find it, or show that it does not exist.

p. 139 Exercise 1.5.24 The M should be n: "How large does n have to be..."

p. 140 Exercise 1.5.25 should read: "For the following functions, can you choose a value for f at $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ to make the function continuous?"

p. 140 Exercise 1.6.6 part (b) should read: "Every polynomial over the real numbers can be factored into real linear factors and real quadratic factors with complex roots."

p. 142 Exercise 1.7.14 like Exercise 1.7.13 part b.

p. 143 Exercise 1.7.16 part (a) should be with the exercises for Section 1.8.

p. 144 Exercise 1.8.2 should read for $t > 1$ not $s > 1$

p.144 Exercise 1.8.4 differentiable, not smooth

p. 145 Exercise 1.8.9: $[D\mathbf{f}(\mathbf{0})]$ should be $[\mathbf{Df}(\mathbf{0})]$ and $\mathbf{g} \circ \mathbf{f}(\mathbf{x})$ should be $(\mathbf{g} \circ \mathbf{f})(\mathbf{x})$. Part (a) must specify differentiable:

(a) If $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$ is differentiable, and $[D\mathbf{f}(\mathbf{0})]$ is not invertible, then there is no differentiable function $\mathbf{g} : \mathbb{R}^2 \to \mathbb{R}^2$ such that $\mathbf{g} \circ \mathbf{f}(\mathbf{x}) = \mathbf{x}$.

p. 145 Exercise 1.9.2 part (b) should have two stars; it is hard.

p.146 The second margin note: $|\sin x| \leq |x|$ not $|\sin x| \leq x$