

Corrections to the First Printing: Chapter 3

p. 250 first line of the paragraph immediately above Def. 3.3.1, $X \subset \mathbb{R}^2$, not $X \in \mathbb{R}^2$: “It says that a subset $X \subset \mathbb{R}^2$ is a smooth curve ... ”

pp. 258–259: In Example 3.1.15, U should be D , to be consistent with the notation in Exercise 3.1.5.

p. 258, Equation 3.1.13 should be

$$D_{x,y} = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \text{ such that } x^2 + y^2 < 1 \right\}$$

p. 264, Definition 3.1.20, a smooth curve $C \subset \mathbb{R}^n$ not $C \in \mathbb{R}^n$.

p. 265 Definition 3.1.21, “ $S \subset \mathbb{R}^3$ is a smooth mapping,” not $S \in \mathbb{R}^3$

p. 266, Second line of Section 3.2, $X \subset \mathbb{R}^2$ not $X \in \mathbb{R}^2$

p. 267 Equation 3.2.1 should read

$$X_2 = \text{the set of all } (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) \in (\mathbb{R}^2)^4 \text{ such that } \dots$$

One could also write

$$X_2 = \{(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) \in (\mathbb{R}^2)^4\}.$$

p. 268 First margin note: we tried this with string and a straw from out kitchen drawer, and the results were misleading; the string was too elastic and the pieces of straw took up too much room to be good models of “universal joints.”

p. 270, last margin note: $\mathbf{u} + \mathbf{g}(\mathbf{u})$ is a point of the graph of $\mathbf{g}$, not a point of the graph of $\mathbf{u}$.

p. 272, Definition 3.2.5, (1): U is an open subset of $\mathbb{R}^k$, not of $\mathbb{R}^n$.

p. 277, last margin note: when we wrote

$$\sum_{I \in \mathcal{I}_n} a_I \mathbf{x}^I.$$

we should have specified that $\deg I \leq k$.

p. 280, last line of Proposition 3.3.12: “Then for any particular $I \in \mathcal{I}_n \dots$ ” ($\mathcal{I}_n$, not $\mathcal{I}$, which is undefined).

p. 281, second-last line, “If $I > J$, the result is also 0” should be “If any index of I is greater than the corresponding index of J , the result is also 0.” (undefined notation)

p. 284 We should have mentioned here that Proposition 3.3.19 is proved in Appendix A.7.

p. 285 In Equation 3.3.41, the expression in the overbrace should be $p = Q_{f,\mathbf{a}}^k \vec{\mathbf{h}}^J$, not $p = Q_{f,\mathbf{a}}^k$.

p. 287, in Proposition 3.4.2 and elsewhere, $\log$ means natural logarithm (log base e). Because this appears to lead to some confusion, in future printings it will be replaced by $\ln$, and $\log$ will only mean the “common logarithm,” log base 10.

p. 287 In Equations 3.4.3 through 3.4.7, all the $o(x)$ should be $o(|x|)$. In Equation 3.4.6, $o(x^{n+1})$ should be $o(|x|^n)$.

p. 287 Propositions 3.4.3 and 3.4.4 are proved in Appendix A.8.

p. 289, Equation 3.4.14, the third term should be preceded by a minus sign:

$$-\frac{f''(a)u^2 + f''(b)v^2}{2(f(a) + f(b))^2}$$

p. 289 In Theorem 3.4.7, F should be $\mathbf{F}$, and the dimensions of the domain and range of the Taylor polynomial for $\mathbf{g}$ are wrong; it goes from $\mathbb{R}^m$ to $\mathbb{R}^n$. In future printings the theorem will read:

Theorem 3.4.7 (Taylor polynomials of implicit functions) If $\mathbf{F}$ is of class C^k for some $k \geq 1$, then the implicit function $\mathbf{g}$ is also of class C^k , and its Taylor polynomial of degree k , $P_{\mathbf{g}, \mathbf{b}}^k : \mathbb{R}^m \rightarrow \mathbb{R}^n$, satisfies

$$P_{F, \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}}^k \left(\begin{pmatrix} P_{\mathbf{g}, \mathbf{b}}^k(\mathbf{b} + \mathbf{u}) \\ \mathbf{b} + \mathbf{u} \end{pmatrix} \right) \in o(|\mathbf{u}|^k).$$

It is the unique polynomial map of degree at most k that does so.

No proof is given of this theorem. The difficult part is proving that the implicit function is k times differentiable. We do not plan to add a proof in future printings/editions, but we do plan to find and give a reference.

p. 290 In the equation after Equation 3.4.16, we left out the exponent corresponding to the 3 in xyz^3 ; the equation should be

$$(1+u)^2 + (1+v)^3 + \left((1+u)(1+v) \left(1 + a_{1,1}u + a_{2,1}v + \frac{a_{1,1}}{2}u^2 + a_{1,2}uv + \frac{a_{2,2}}{2}v^2 \right)^3 \right) - 3 \in o(u^2 + v^2).$$

p. 292 In the second line of Theorem 3.5.3, “ m linearly independent linear functions” should be “ $m = k + l$ linearly independent linear functions.”

p. 293, between Equations 3.5.10 and 3.5.11, “... we see that $B = 2y - 4z$ will allow us to complete the square,” not $B = y - 2z$

p. 297 In the first line, a should be α : For any $\vec{\mathbf{v}} \in \ker T$, the terms $\alpha_{k+1}(\vec{\mathbf{v}}), \dots, \alpha_{k+l}(\vec{\mathbf{v}})$ of $Q(\vec{\mathbf{v}})$ vanish, so ...

p.301 Theorem 3.6.6, last line: is not a local maximum.

p.302 There are three errors in the proof of Theorem 3.6.8, one substantive. It is “Proposition 3.5.11” not Theorem 3.5.11. Equation 3.6.11 is wrong; it should be

$$\frac{f(\mathbf{a} + t\vec{\mathbf{h}}) - f(\mathbf{a})}{t^2} = \frac{t^2 Q(\vec{\mathbf{h}}) + r(t\vec{\mathbf{h}})}{t^2} \geq C|\vec{\mathbf{h}}|^2 + \frac{r(t\vec{\mathbf{h}})}{t^2}.$$

In the last line of the proof, $f(\mathbf{a})$ should be $f(\mathbf{a})$.

p. 303, third line: “But if the quadratic form is degenerate . . . ,” not “But if the form is degenerate”

p. 309 line immediately before Equation 3.7.7: replace “maximum” by “critical point.”

p. 310 There should be a margin note at the top of the page that says

The value $\lambda = \sqrt{\frac{3}{8}}$ gives the maximum; $\lambda = -\sqrt{\frac{3}{8}}$ gives the minimum, $x + y = -\sqrt{3/2}$. Since the constraint manifold is an ellipse, there can be no saddle points.

p. 320 In Definition 3.8.3, γ , not *gamma*.

p. 324 Equation 3.8.34 should have minus sign:

$$H = \frac{-1}{2(1+c^2)^{3/2}}(a_{2,0}(1+a_2^2) - 2a_1a_2a_{1,1} + a_{0,2}(1+a_1^2)).$$

p. 327 In Equation 3.8.47, some parentheses are in the wrong place. it should be

$$\begin{aligned} Z = & \frac{1}{2} \left(\overbrace{-\frac{1}{c^2\sqrt{1+c^2}} \left(a_{2,0}a_2^2 - 2a_{1,1}a_1a_2 + a_{0,2}a_1^2 \right)}^{A_{2,0}} \right) X^2 \\ & + 2\frac{1}{c^2(1+c^2)} \left(-a_{2,0}a_1a_2 - a_{1,1}a_2^2 + a_{1,1}a_1^2 + a_{0,2}a_1a_2 \right) XY \\ & - \frac{1}{c^2(1+c^2)^{3/2}} \left(a_{2,0}a_1^2 + 2a_{1,1}a_1a_2 + a_{0,2}a_2^2 \right) Y^2 \Big) + \dots \end{aligned}$$

p. 330 Proposition 3.8.12 might be clearer written as follows;

The vectors $\vec{\mathbf{t}}(0), \vec{\mathbf{n}}(0), \vec{\mathbf{b}}(0)$ form the orthonormal basis (Frenet frame) with respect to which our adapted coordinates are computed. Thus the point with coordinates X, Y, Z in the new, adapted coordinates is the point

$$\mathbf{a} + X\vec{\mathbf{t}}(0) + Y\vec{\mathbf{n}}(0) + Z\vec{\mathbf{b}}(0)$$

in the old x, y, z coordinates.

(Note that Equation 3.8.57 says that the point $\mathbf{a}$ in the old coordinates is the point $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ in the new coordinates, which is what we want.)

p. 330: In future printings we will add the following margin note next to Proposition 3.8.13:

The derivatives $\vec{\mathbf{t}}'$, $\vec{\mathbf{n}}'$, and $\vec{\mathbf{b}}'$ are computed with respect to arc length. Think of the point $\mathbf{a}$ as the place where you set the odometer reading to 0, accounting for $\vec{\mathbf{t}}'(0)$, etc. The derivatives are easiest to compute at 0, but the equations $\vec{\mathbf{t}}'(0) = \kappa\vec{\mathbf{n}}(0)$ and so on are true at any point (any odometer reading).

p. 330, Proposition 3.8.14, the curve is parametrized by $\gamma : \mathbb{R} \rightarrow \mathbb{R}^3$, not by $\gamma : \mathbb{R}^3 \rightarrow \mathbb{R}$, and $\kappa(t)$ should be $\kappa(\gamma(t))$. In Proposition 3.8.15, $\tau(t)$ should be $\tau(\gamma(t))$. In future printings there will be a note:

Many texts refer to $\kappa(t)$ and $\tau(t)$, including this one in earlier printings. However, curvature and torsion are invariants of a curve: they depend only on the point at which they are evaluated. Strictly speaking, it does not make sense to think of them as functions of time.

p. 331 In Example 3.8.16, $\kappa(t)$ should be $\kappa(\gamma(t))$ and $\tau(t)$ should be $\tau(\gamma(t))$. A note has been added to the page:

Strictly speaking, $\vec{t}$, $\vec{n}$, $\vec{b}$, κ , and τ should be considered as functions of $\delta(s(t))$: the point where the car is at a particular odometer reading, which itself depends on time. However, the notation is already fearsome, and we hesitate to make it any more difficult than it is.

p. 332 A note has been added next to Equation 3.8.67:

The derivative κ' is the derivative of κ with respect to arc length.

p. 334 Exercise 3.1.12, part (c) deserves a star or perhaps two. Our solution uses material in Exercise 3.1.20.

p. 335, Exercise 3.1.15: the function in the displayed equation should be F , not f .

p. 336 Exercise 3.1.18, part (e): the set of non-invertible symmetric 2×2 matrices, not the set of non-invertible 2×2 matrices

p. 337, Exercise 3.2.2, last line of part (a), third coordinate of point is z_2 not z_3

p. 337, Exercise 3.2.3.: There is an error in this problem. It should say “... and greater than $|l_1 - l_2|$ and $|l_3 - l_4|$.”

p. 337, Exercise 3.2.6: the words “In Example 3.2.1” should be deleted.

p. 338, Exercise 3.28, part (d): $A^\top A - 1$, not $AA^\top - 1$ (to be consistent with parts (a) and (c)).

p. 339 Exercise 3.3.3, part (d): Theorem 3.3.9, not Proposition 3.3.11.

p. 340 Exercise 3.3.7: part (c) should have two stars.

p. 340, Exercise 3.3.8, in the displayed equation, $f^k(a)$ should be $f^{(k)}(a)$.

p. 340, Exercise 3.3.13, “so that $f(1) = e$, not $f(0) = e$.”

p. 341 Exercise 3.3.16: “What condition must the coefficients of p satisfy,” not “What condition must the coefficients of f satisfy.”

p. 342 Exercise 3.5.1 part (d), the last sentence should be:

“Show that $B(\Phi_p(\mathbf{a}), \Phi_p(\mathbf{b}))$ is a symmetric bilinear function on $\mathbb{R}^n \dots$ ”

p. 342, Exercise 3.5.2, 1st sentence, “denote by $Q_B : V \rightarrow \mathbb{R}$ the function $Q_B(\underline{\mathbf{v}}) = B(\underline{\mathbf{v}}, \underline{\mathbf{v}})$ ” (not “denote by $Q_B : V \rightarrow \mathbb{R}$ the function $Q(\underline{\mathbf{v}}) = B(\underline{\mathbf{v}}, \underline{\mathbf{v}})$ ”). Exercise 3.4.5 (b) should be in Section 3.6, where critical points are discussed.

p. 343, Exercise 3.5.5: the integral lacks dt . The equation should read

$$Q(p) = \int_0^1 (p(t))^2 - (p'(t))^2 dt.$$

p.343, Exercise 3.5.6 (b), the displayed equation should have more parentheses, making it

$$x(x-1)\dots(x-(j-1))(x-(j+1))\dots(x-k),$$

p. 344, Exercise 3.5.13 was poorly stated. It should read:

Identify $\begin{pmatrix} a \\ b \\ d \end{pmatrix} \in \mathbb{R}^3$ with the upper triangular matrix $M = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$.

(a) What kind of surface in $\mathbb{R}^3$ do you get by setting $\text{tr}(M^2) = 1$?

(b) What kind of surface in $\mathbb{R}^3$ do you get by setting $\text{tr}(MM^T) = 1$?

p. 344 Exercise 3.5.17 should read:

Consider the vector space of Hermitian 2×2 matrices:

$H = \begin{bmatrix} a & b+ic \\ b-ic & d \end{bmatrix}$, where a, b, c, d are real numbers. What is the signature of the quadratic form $Q(H) = \det H$?

p. 344, Exercise 3.5.18, the introduction now reads:

Identify and sketch the conic sections and quadratic surfaces of equation $Q(\mathbf{x}) = 1$ when $Q(\mathbf{x})$ is one of the quadratic forms defined by the following matrices:

p. 345 Exercise 3.5.19 was not stated clearly. It should read:

For each of the following equations, determine the signature of the quadratic form represented by the left-hand side. Where possible, sketch the curve or surface represented by the equation.

p. 346 Exercise 3.7.3, part (a): The equation should read

$$\varphi \begin{pmatrix} x \\ y \\ z \end{pmatrix} = ax + by + cz - 1 = 0.$$

In addition, there should be a semicolon before the “i.e.”

Exercise 3.7.6: Throughout the exercise, replace $AA^\top$ by $A^\top A$.

p. 347 Exercise 3.7.7 In the second printing, this exercise will read:

(a) Find the critical points of the function $x^3 + y^3 + z^3$ on the intersection of the planes of equation $x + y + z = 2$ and $x + y - z = 3$.

(b) Are the critical points maxima, minima, or neither?

p. 347 Exercise 3.7.11 is identical to Exercise 3.7.7.

p. 347 Exercise 3.7.13 misstated. It should read: (a) Find the critical points of xyz , if x, y, z belong to the surface of equation $x + y + z^2 = 16$.

(b) Show that the unique critical point where x, y, z are all ≥ 0 is a maximum.

p. 347 Exercise 3.7.14 should be in Chapter 4, after multiple integrals are discussed in Section 4.5.

p. 349 Exercise 3.8.4 is badly stated. It should read:

(a) Show that the equation $y \cos z = x \sin z$ expresses z implicitly as a function $z = g_r \left(\begin{smallmatrix} x \\ y \end{smallmatrix} \right)$ near the point $\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$.

(b) Show that $D_1 g_r \begin{pmatrix} r \\ 0 \end{pmatrix} = D_1^2 g_r \begin{pmatrix} r \\ 0 \end{pmatrix} = 0$. Hint: The x -axis is contained in the surface.

p. 349 Exercise 3.8.5 should ask for both the Gaussian and the mean curvature.

p. 349 Exercise 3.8.8, part (b), should read

Show that

$$K(x) = \frac{-f''(x)}{f(x) \left(1 + (f'(x))^2 \right)^2}.$$

(In the denominator, $f'(x)$ should be $(f'(x))^2$.)

p. 349 Exercise 3.8.10 does not need a star.