Corrections to the First Printing: Chapter 4

p. 358 Equation 4.1.23 should be

$$f(x) = x\chi_{[0,1)}(x); \quad \text{i.e.,} \quad [0,1) \text{ not } [0,1]$$

p. 358, the last margin note should read:

Since we are in dimension 1, our cubes are intervals:

$$C_{k,N} = \left[\frac{k}{2^N}, \frac{k+1}{2^N}\right).$$

p. 360, the proof of Proposition 4.1.12 should begin with the sentence, "We will prove this in the case where f_1 and f_2 are nonnegative." A margin note should read, "The general case of Proposition 4.1.12 can be proved using $f_1 = f_1^+ + f_1^-$, $f_2 = f_2^+ + f_2^-$ (see Definition 4.3.4)." When time permits, this proof will be added to the web page.

p. 368, Equation 4.2.13, the integrals are over $\mathbb{R}$, not R.

p. 370, Equation 4.2.18: the exponent needs a minus sign. The normal distribution is given by

$$\mu(x) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}.$$

p. 370 In Theorem 4.2.11, "expectation value" should be "expectation" (or "expected value").

p. 371 Equation 4.2.23 in the numerator, $\sqrt{1000}$, not $2\sqrt{1000}$

p. 372 Second line: within two standard deviations of the mean, not one; as a result, we want $\frac{1}{2}\%$ to be the standard deviation, and Equation 4.2.25 should be

$$\frac{1}{2\sqrt{n}} = \frac{1}{200}; \quad \text{i.e.,} \quad n = 10000.$$

The answer in the footnote should be 2500, not 625.

p. 377 In the second margin note, it should be " ... we will be able to drop the requirement that X be compact," not "that M be compact."

p. 380, Equation 4.3.18 should be

$$y = +\sqrt{1-x^2} \quad \text{and} \quad y = -\sqrt{1-x^2}.$$

p. 381, in Definition 4.4.1, "$X \subset \mathbb{R}^n$," not $X \in \mathbb{R}^n$, and "$X \subset \cup B_i$," not $X \in \cup B_i$."

p. 383 The caption to Figure 4.4.2 should read: The trajectory with slope 2/5, at center, visits more of the square than the trajectory with slope 1/2, at left. The slope of the trajectory at right closely approximates an irrational number; if allowed to continue, this trajectory would visit nearly every part of the square. (In practice, the drawing would soon become black.)

p. 384 Equation 4.4.4 should have $\subset$, not $\in$: $\Delta \subset \cup B_i$.

p. 394, The caption to Figure 4.5.10 should read: The situation if the first dart lands above the diagonal. If the second dart lands in the shaded area, then the determinant is negative because the second vector is clockwise from the first (Proposition 1.4.14). For values of x_2 to the right of the vertical dotted line (which has x-coordinate x_1), y_2 goes from 0 to 1. For values of x_2 to the left of the vertical dotted line, we must consider separately values of y_2 from 0 to $\frac{y_1 x_2}{x_1}$ (the shaded region, with $-\det$) and values of y_2 from $\frac{y_1 x_2}{x_1}$ to 1 (the unshaded region, with $+\det$).

p. 394 Strictly speaking, when we compute the first term of the inner integral, it should be

$$-\overbrace{0}^{\text{eval. at } y_2=0} \qquad \text{not } +0.$$

p. 394, footnote 7 is wrong. It should read:

Exercise 4.5.7 asks you to compute the integral that way. The integral is then the sum of the integral for the half of the square below the diagonal (given by Equation 4.5.25), and the integral for the half above the diagonal. The latter is

$$\int_0^1 \int_{x_1}^1 \int_0^{\frac{x_1}{y_1}} \overbrace{\left(\int_0^{\frac{y_1 x_2}{x_1}} \underbrace{(x_2 y_1 - x_1 y_2)}_{-\det} \, dy_2 + \int_{\frac{y_1 x_2}{x_1}}^1 \underbrace{(x_1 y_2 - x_2 y_1)}_{+\det} \, dy_2 \right)}^{\text{for } x_2 \text{ to the left of the vertical line with } x\text{-coordinate } x_1} dx_2 \, dy_1 \, dx_1$$

$$+ \quad \int_0^1 \int_{x_1}^1 \int_{\frac{x_1}{y_1}}^1 \underbrace{\left(\int_0^1 (x_2 y_1 - x_1 y_2) \, dy_2 \right)}_{\substack{\text{for } x_2 \text{ to the right of the vertical line} \\ \text{with } x\text{-coordinate } x_1;\ -\det}} dx_2 \, dy_1 \, dx_1.$$

p. 395 In the underbrace for the third line of Equation 4.5.27, $y_1^2 4$ should be $\frac{y_1^2}{4}$.

p. 399 In $\int_0^\infty f(x) e^{-x}\, dx$, the function $f(x)$ is weighted by e^{-x}; when we speak of "functions with a particular weight" we are not referring to the weights of Guassian rules or Simpson's method.

p. 400: In the second line of Equation 4.6.16, the dx_i should be omitted; that line should read

$$= \left(\sum w_i f_1(p_i) \right) \dots \left(\sum w_i f_n(p_i) \right)$$

p. 401: The left-hand side of Equation 4.6.17 should read

$$\int_{[-1,1]^n} f(\mathbf{x}) \, |d^n \mathbf{x}|.$$

(The integral is a multiple integral, over the region in $\mathbb{R}^n$ where all the coordinates are in [-1,1].)

p. 401 In the margin note, it should be $w_1 = w_3$, not $w_1 = w_2$; i.e., the one-dimensional weights are

$$w_1 = w_3 = \frac{b-a}{6n} \cdot 1 = \frac{1}{3} \quad \text{and} \quad w_2 = \frac{1}{3}(4) = \frac{4}{3}.$$

p. 402 In Definition 4.6.7, part (3)

$$\overline{s}^2 = \frac{1}{N}\sum_{i=1}^{N} a_i - \overline{a}^2 \quad \text{should be} \quad \overline{s}^2 = \frac{1}{N}\sum_{i=1}^{N}(a_i - \overline{a})^2$$

p. 403 In the bottom margin note, $\mathbf{x}_9$ should be x_9 (not bold).

p. 404 Definition 4.7.3 has been changed. As written, the boundary of a paving does not include the border between a tile P and $\mathbb{R}^n - X$. We have added a definition of "boundary" to Section 1.5:

The boundary of a subset $A \subset \mathbb{R}^n$ is those points for which every neighborhood intersects both A and the complement of A. It is also the intersection of the closure of A and the closure of the complement of A.

and have changed Definition 4.7.3 to read

The boundary $\partial\mathcal{P}$ of $\mathcal{P}$ is the union of the boundaries of the tiles:

$$\partial\mathcal{P} = \bigcup_{P\in\mathcal{P}} \partial P.$$

p. 404 last margin note, "The set of all $P \in \mathcal{P}$ completely paves $X \subset \mathbb{R}^n$," not "The set of all $P \in \mathcal{P}$ completely paves $\mathbb{R}^n$."

p. 408, clarification, not correction: top of the page: we define $A_{i,j}$ but in Equation 4.8.9 we have the case where $j = 1$: $A_{i,1}$, the $(n-1) \times (n-1)$ matrix obtained from A by erasing the ith row and the first column.

p. 409 next to last line: Section 2.3, not Section 2.1

p. 413, third line: it would be better to replace "so $\det E = \det I$" by "so $\det E = 1$." Fifth line: replace "$\det E^\top = \det I$" by "$\det E^\top = 1$."

p. 414 Since we discuss the change of basis matrix only in volume two, it would be better not to mention it in Theorem 4.8.13. That theorem, and the preceding paragraph, should be:

The following theorem acquires its real significance in the context of abstract vector spaces, where we will see that it means that the determinant function is basis independent. This is discussed in volume two; in this volume we will find the theorem useful in proving Corollary 4.8.22.

Theorem 4.8.13. *If P is invertible, then*

$$\det A = \det(P^{-1}AP).$$

p. 416 Theorem 4.8.18: it is probably obvious, but we should have said that $\mathrm{Perm}(1, \dots, n)$ denotes the set of permutations of $1, \dots, n$.

p. 417 Right after the table, "So $\det A = 45 + 84 + 96 - 48 - 72 - 105 = 0$" ($-48$, not -42).

p. 419 In the proof of part (b), the word "term" may not be clear. The beginning of that proof should read:

... Put another way, we want to find the terms that are linear in h of the expansion given by Equation 4.8.46 for

$$\det(I + hB) = \det \begin{bmatrix} 1 + hb_{1,1} & hb_{1,2} & \dots & hb_{1,n} \\ hb_{2,1} & 1 + hb_{2,2} & \dots & hb_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ hb_{n,1} & hb_{n,2} & \dots & 1 + hb_{n,n} \end{bmatrix}.$$

Equation 4.8.56 shows ... .

p. 419 In the bottom margin note, the matrix in the second line of the equation should be preceded by det. In Equation 4.8.57, if you have trouble getting from the next-to-last line to the last line, replace the 1 in the next-to-last line by $\det I$; the limit in that line is then the directional derivative of det at I in the direction $A^{-1}B$, which by Proposition 1.7.12 can be written $[\mathbf{D}\det(I)](A^{-1}B)$, which by part (b) of the theorem is $\mathrm{tr}(A^{-1}B)$.

p. 421 The last margin note should say "The first requirement for a paving of $\mathbb{R}^n$," not "The first requirement for a paving."

p. 426 Although it makes for a lot of parentheses, the sum in the last line of Equation 4.9.19 should perhaps be written

$$\sum_{P \in (T(\mathcal{D}_N(\mathbb{R}^n)))};$$

we are summing over those P that are in the decomposition $T(\mathcal{D}_N(\mathbb{R}^n))$.

p. 426, Definition 4.9.8: "parallelepiped" should be "k-parallelogram."

p. 427 The margin note should be changed, since this issue is addressed only in the sequel to the present volume. It should read:

The meaning of expressions like du is explored in Chapter 6. In volume two we will see that we can use the change of variables formula in higher dimensions without requiring exact correspondence of domains, using the language of forms. This is what you were using (more or less blindly) in one dimension.

p. 430, caption to Figure 4.10.4: φ is latitude, not longitude.

p. 431 The third margin note is wrong. It should be "At $\varphi = -\pi/2$ and $\varphi = \pi/2$, $r = 0$." Two lines before Definition 4.10.9, "projection in" should be "projection onto." In the third line of Definition 4.10.9, "the projection" should be "its projection."

p. 432 The last margin note should be deleted; there is no V in the theorem. We have changed names of variables in in Equation 4.10.21; it now reads

$$\int_Y f(\mathbf{y})\, |d^n\mathbf{y}| = \int_X (f \circ \Phi)(\mathbf{x}) \left|\det[\mathbf{D}\Phi(\mathbf{x})]\right| |d^n\mathbf{x}|.$$

p. 433 Three lines before Equation 4.10.4, "since $\det[DP] = r$" not "since $\det[D\Phi] = r$."

p. 435: The determinant of the Jacobian is missing in Equation 4.10.31. The correct equation is

$$\int_V f(\mathbf{v})\,|d^n\mathbf{v}| = \int_U (f \circ \Phi)(\mathbf{u})\big|\det[\mathbf{D}\Phi(\mathbf{u})]\big||d^n\mathbf{u}|.$$

p. 440 Proposition 4.11.7, $M \subset \mathbb{R}^n$ not $M \in \mathbb{R}^n$. The title may be misleading; we have changed it and part (a) to read

Proposition 4.11.7 (Manifold of lower dimension has volume 0) (a) Any manifold $M \subset \mathbb{R}^n$ that is a closed subset of $\mathbb{R}^n$ and of dimension less than n has n-dimensional volume 0.

p. 441 First margin note: K, not N, to be consistent with the notation of Definition 4.11.9.

p. 443 In Equation 4.11.30, right-hand side, it should be sup_k, not $\sup_R$.

p. 443 In the second margin note, Equation 4.11.29 should be 4.11.30.

p. 445 Clarification, not correction: We get π in the numerator in Equation 4.11.3 because

$$\int \frac{1}{1+u^2}\,du = \arctan u;$$

$$\lim_{u\to\pm\infty} \arctan u = \pm\frac{\pi}{2}.$$

p. 445, in the line after Equation 4.11.43, $1/2x$ should have been written $\frac{1}{2x}$ or $1/(2x)$.

p. 446, Theorem 4.11.16: in the line immediately before Equation 4.11.45, $(f \circ \varphi)|\det[D\Phi]|$ should be $(f \circ \Phi)|\det[\mathbf{D}\Phi]|$. In Equation 4.11.45 an absolute value sign is misplaced; the equation should be

$$\int_V f(\mathbf{v})|d^n\mathbf{v}| = \int_U (f \circ \Phi)(\mathbf{u})\,\big|\det[\mathbf{D}\Phi(\mathbf{u})]\big|\,|d^n\mathbf{u}|.$$

p. 446 in Equation 4.11.46, the sum is over $\mathrm{vol}_n\, C$:

$$\sum_{\substack{C\in\mathcal{D}_N(\mathbb{R}^n),\\ \overline{C}\cap\partial U_R\neq\phi}} \mathrm{vol}_n\, C < \frac{1}{R},$$

p. 448: in the fourth line of text, "with respect to a different integral" should be "with respect to a different variable." The title of Theorem 4.11.18 has been changed to "Differentiating under the integral sign." In Theorem 4.11.18, "except perhaps on a set of $\mathbf{x}$ of volume 0" should be "except perhaps for $\mathbf{x}$ in a set of volume 0." In Equation 4.11.59, it might be clearer to have the $|d^n\mathbf{x}|$ not part of the fractions, and the D_1 should be replaced by D_t to be consistent with the notation of the theorem:

$$\begin{aligned} DF(t) &= \lim_{h\to 0}\frac{F(t+h)-F(t)}{h} = \lim_{h\to 0}\int_{\mathbb{R}^n}\frac{f(t+h,\mathbf{x})-f(t,\mathbf{x})}{h}|d^n\mathbf{x}| \\ &= \int_{\mathbb{R}^n}\lim_{h\to 0}\frac{f(t+h,\mathbf{x})-f(t,\mathbf{x})}{h}|d^n\mathbf{x}| = \int_{\mathbb{R}^n} D_t f(t,\mathbf{x})\,|d^n\mathbf{x}|. \end{aligned}$$

p. 449: Equations 4.11.62 and 4.11.63 contain errors, and Laplace transforms are not discussed. The equation in the line immediately after Equation 4.11.63 also contains an error; $|e^i a - 1|$ should be $|e^{ia} - 1|$ The text on p. 449 should be replaced by the following:

This function $\hat{f}$ is called the *Fourier transform* of f. Passing from f to $\hat{f}$ is one of the central constructions of mathematical analysis; many entire books are written about it. We want to use it as an example of differentiation under the integral sign.

According to Theorem 4.11.18,

$$D\hat{f}(\xi) = \int_{\mathbb{R}} D_\xi\big(f(x)e^{ix\xi}\big)\, dx = \int_{\mathbb{R}} f(x) D_\xi(e^{ix\xi})\, dx = \int_{\mathbb{R}} ixf(x)e^{ix\xi}\, dx = \widehat{ixf}(\xi),$$

provided that the difference quotients

$$\left|\frac{e^{i(\xi+h)x} - e^{i\xi x}}{h} f(x)\right| = \left|\frac{e^{ihx} - 1}{h}\right| |f(x)|$$

are all bounded by a single integrable function. Since $|e^{ia} - 1| = 2|\sin(a/2)| \leq |a|$ for any real number a, we see that this is satisfied if $xf(x)$ is an I-integrable function.

More generally, if $x^p f(x)$ is an I-integrable function, then $\hat{f}$ is p times differentiable, and

$$D^p \hat{f}(\xi) = \int_{\mathbb{R}} D_\xi^p \left(e^{ix\xi} f(x)\right)\, dx = \int_{\mathbb{R}} (ix)^p f(x) e^{ix\xi}\, dx = \widehat{(ix)^p f}(\xi).$$

In other words, the faster f decreases at infinity, the smoother the Fourier transform $\hat{f}$ is. This brings out one feature of the Fourier transform: growth conditions on the original function f get translated into smoothness conditions for the Fourier transform.

Rather than differentiating the Fourier transform, we might want to Fourier transform the derivative, which we can do if both f and f' are I-integrable. This is best done by integration by parts:

$$\begin{aligned}\widehat{(f')}(\xi) &= \int_{\mathbb{R}} f'(x) e^{i\xi x} dx = \lim_{A\to\infty} \int_{-A}^{A} f'(x) e^{i\xi x} dx \\ &= \lim_{A\to\infty} \left[f(x)e^{i\xi x}\right]_{-A}^{A} - \lim_{A\to\infty} \int_{-A}^{A} i\xi f(x) e^{i\xi x} dx.\end{aligned}$$

Since f and f' are I-integrable, then $\lim_{x\to\pm\infty} f(x) = 0$, so

$$\lim_{A\to\infty} \left[f(x)e^{i\xi x}\right]_{-A}^{A} = 0,$$

(You might think that if f is I-integrable, then f must tend to 0 at infinity. This isn't true; for instance f could have spikes of height 1 and width $1/n^2$ at all the integers. But if f' is also I-integrable, then f does have to tend to 0 at infinity.) So

$$\widehat{(f')}(\xi) = -i\xi \int_{\mathbb{R}} f(x) e^{i\xi x} dx = -i\xi \hat{f}(\xi).$$

This shows another feature of the Fourier transform: it turns differentiation into multiplication. For instance, the Fourier transform of the differential equation

$$a_p D^p f + \cdots + a_0 f = g \quad \text{is} \quad \big(a_p(-i\xi)^p + a_{p-1}(-i\xi)^{p-1} + \cdots + a_0\big)\hat{f} = \hat{g},$$

which gives

$$\hat{f} = \frac{\hat{g}}{(-i\xi)^p a_p + \cdots + a_0}.$$

This gives $\hat{f}$, and if you know how to undo the Fourier transform, it gives f. This ability to change the analytic operation of differentiation into the algebraic operation of multiplication is one important reason why Fourier series are essential in the theory of differential equations, especially partial differential equations.

The *Laplace transform* is another important transform. The Laplace transform $\mathcal{L}(f)$ of f is defined by the formula

$$\mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st}dt.$$

(Note that the integral is from 0 to ∞, not $-\infty$ to ∞.) Depending on the range of values of s you are interested in, the Laplace transform $\mathcal{L}(f)$ exists for quite a broad range of functions f. For instance, it is a continuous function of $s \in [0,\infty)$ if f is I-integrable, and it is defined and continuous on $(0,\infty)$ if f grows more slowly than some polynomial.

Again, under appropriate circumstances, we can differentiate under the integral sign:

$$D(\mathcal{L}f)(s) = \int_0^\infty D_s(f(t)e^{-st})dt = \int_0^\infty -tf(t)e^{-st}dt = \big(\mathcal{L}(-tf)\big)(s).$$

If f grows more slowly than some polynomial, and $s \in (0,\infty)$, then Equation 4.11.57 tells us that this differentiation under the integral sign is justified. Indeed, the family of functions

$$\frac{e^{-(s+h)t} - e^{-st}}{h} f(t) = e^{-st} f(t) \frac{e^{-ht} - 1}{h}$$

is then bounded by $e^{-st}|tf(t)|$, which is I-integrable.

As in the case of the Fourier transform, there is much more to do with the Laplace transform, but it is beyond the scope of this book.

p. 451 Exercise 4.1.12: in the displayed equation, it should be 2^{nN} on the right-hand side, not 2^N:

$$\int_{\mathbb{R}^n} D_{2^N} f(\mathbf{x}) |d^n\mathbf{x}| = 2^{nN} \int_{\mathbb{R}^n} f(\mathbf{x})\, |d^n\mathbf{x}|.$$

Part (b) is independent of part (a) and should be a separate exercise.

p. 451 Exercise 4.1.11 belongs with the exercises for Section 4.2. In addition, in the displayed equation of part (b), the (A) in the denominator should be $M(A)$.

p. 451 Exercise 4.1.12: "dilation of a function by a of a function" should be "dilation by a of a function."

p. 451, Exercise 4.1.16 part (a) is badly stated and should be ignored.

p. 451 first line of Exercise 4.1.12 part (b): dyadic cubes, not canonical cubes

p.452, Exercise 4.1.17, the equations after "Show that if $f : \mathbb{R}^n \to \mathbb{R}$ is integrable, then" should read as follows (upper and lower integrals, not upper and lower sums, so $\overset{\circ}{U}(f)$ not $\overset{\circ}{U}_N(f)$, and so on):

$$\overset{\circ}{U}(f) = \lim_{N\to\infty} \sum_{C\in\mathcal{D}_N(\mathbb{R}^n)} M_{\overset{\circ}{C}}(f)\,\mathrm{vol}_n(C)$$

$$\overset{\circ}{L}(f) = \lim_{N\to\infty} \sum_{C\in\mathcal{D}_N(\mathbb{R}^n)} m_{\overset{\circ}{C}}(f)\,\mathrm{vol}_n(C)$$

$$\overline{U}(f) = \lim_{N\to\infty} \sum_{C\in\mathcal{D}_N(\mathbb{R}^n)} M_{\overline{C}}(f)\,\mathrm{vol}_n(C)$$

$$\overline{L}(f) = \lim_{N\to\infty} \sum_{C\in\mathcal{D}_N(\mathbb{R}^n)} m_{\overline{C}}(f)\,\mathrm{vol}_n(C).$$

p. 452 Exercise 4.2.1: The dice are 6-sided.

p. 453 Hint for Exercise 4.3.1: it is Proposition 4.3.6, not Theorem 4.3.6, and $y = \sqrt{1-x^2}$, not $y = \sqrt{x^2-1}$.

p. 453 Exercise 4.3.2, part (a) should begin higher up: "(a) Let $f : \mathbb{R}^n \to \mathbb{R}$ be an integrable function that is symmetric … ". The displayed formula should be

$$\int_{Q^n_{a,b}} f(\mathbf{x})\,|d^n\mathbf{x}| = n! \int_{P^n_{a,b}} f(\mathbf{x})\,|d^n\mathbf{x}|.$$

p. 454 Exercise 4.5.4, part (a) should read: " (a) In the context of Example 4.5.6 (Equation 4.5.21), show that if ..."

p. 455 Exercise 4.4.2 should read "Show that $X \subset \mathbb{R}^n$ has measure 0 if and only if for any $\epsilon > 0$ there exists an infinite sequence of balls … ". ($X \subset \mathbb{R}^n$, not $X \in \mathbb{R}^n$, and addition of "for any $\epsilon > 0$")

p. 455, Exercise 4.5.11: The proof of Theorem 3.3.9 has been relegated to Appendix A.6, so probably you don't recall it. The way the exercise is worded, you might wonder whether we mean to suggest that with a different proof, it is possible to relax the condition that the second partials be continuous. We do not; without that condition, the theorem is false.

p. 456 Exercise 4.5.16 might be clearer stated as follows:

Find the volume of the region $z \ge x^2 + y^2$, $z \le 10 - x^2 - y^2$.

p. 457 Exercise 4.5.17 In the two displayed equations, the integral should be $\int_{Q^n_{0,1}}$, not $\int^n_{Q_{0,1}}$.

p. 458, in Exercise 4.6.4, p is used with two different meanings, and in part (e), e^{-4} should be e^{-x}. In addition, part (d) can be solved using Student MATLAB only for values of m up to 4; for $m = 5$, the professional version of MATLAB is needed.

The exercise should read (a) Find the equations that must be satisfied by points $x_1 < \cdots < x_m$ and weights $w_1 < \cdots < w_m$ so that the equation

$$\int_0^\infty p(x)e^{-x}dx = \sum_{k=1}^m w_k f(x_k)$$

is true for all polynomials p of degree $\leq d$.

(b) For what number d does this lead to as many equations as unknowns?

(c) Solve the system of equations when $m = 1$.

(d) Use Newton's method to solve the system for $m = 2, \dots, 5$.

(e) For each of the degrees above, approximate

$$\int_0^\infty e^{-x} \sin x \, dx \quad \text{and} \quad \int_0^\infty e^{-x} \ln x \, dx.$$

and compare the approximations with the exact values.

p. 458 In Exercise 4.6.5, a dx is missing; the equation should be

$$\int_{-\infty}^\infty p(x)e^{-x^2} \, dx = \sum_{i=0}^k w_i p(x_i)$$

In addition, note that while in Exercise 4.6.4 the integral was from 0 to ∞, here it is from $-\infty$ to ∞.

p. 460 In Exercise 4.8.1, "first column, not "first row." (Rows would work, but the text discusses computing determinants using development by the first column.)

p. 461, Exercise 4.8.6. part (b) should say column operations, not row operations.

p. 462 Exercise 4.9.1 should read

(a) Show that $\Delta(T) = |\det T|$ is the unique mapping $\Delta : \operatorname{Mat}(n,n) \to \mathbb{R}$ that satisfies

(1) For all $T \in \operatorname{Mat}(n,n)$, we have $\Delta(T) \geq 0$.

(2) The function Δ is a symmetric function of the columns (i.e., switching two columns does not change the value).

(3) For all $T = [\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n] \in \operatorname{Mat}(n,n)$, we have

$$\Delta[a\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n] = |a|\Delta[\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n].$$

(4) For all $T = [\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n] \in \operatorname{Mat}(n,n)$, we have

$$\Delta[\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n] = \Delta[\vec{\mathbf{v}}_1 + a\vec{\mathbf{v}}_2, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n].$$

(5) $\Delta(I_n) = 1$.

(b) Show that $T \mapsto \mathrm{vol}_n(T(Q))$ satisfies the properties that characterize Δ.

p. 462 Exercise 4.9.2 should read

Use "dissection" (as suggested in Figure 4.9.2) to prove Equation 4.9.12 when E is type 2.

p. 462, Exercise 4.9.6., there should be a plus sign after $2x_2$:

$$\{\mathbf{x} \in \mathbb{R}^n \mid x_i \geq 0 \text{ for all } i = 1, \dots, n \text{ and } x_1 + 2x_2 + \dots + nx_n \leq n\}?$$

p. 464 Exercise 4.10.7: "What is the volume of the region $Q_A(\vec{\mathbf{x}}) \leq 1$" ($Q_A$, not Q). This exercise should be starred.

p. 464 Exercise 4.10.10 really belongs with exercises for Section 5.2 as it requires the relaxed definition of a parametrization introduced there. Also in Exercise 4.10.10: $0 \leq r < \infty$, not $0 \leq r \leq \infty$.

p. 466 In Exercise 4.11.2, the first line should be "Let $a_n = \frac{(-1)^{n+1}}{n}$" (the exponent is $n+1$, not n.)

p. 466, Exercise 4.11.3, two R in displayed equation should be $\mathbb{R}$:

$$\lim_{k\to\infty} \lim_{R\to\infty} \int_{\mathbb{R}} [f_k(x)]_R \, dx \neq \lim_{R\to\infty} \lim_{k\to\infty} \int_{\mathbb{R}} [f_k(x)]_R \, dx.$$

p. 467, Exercise 4.11.5: the (a) should be deleted as there is no part (b).