

Corrections to the First Printing: Chapter 5

p. 471 Note that $T^\top T$ is symmetric; this makes computing it easier. The margin note starting “Note that one consequence . . . ” would go better on the next page, near Theorem 5.1.4, since while in Proposition 5.1.2 T must be square, in Theorem 5.1.4 there is no such restriction.

p. 471. Equation 5.1.3: there are two errors in the bottom row of the matrix $T^\top T$. The second entry should be $\vec{v}_2 \cdot \vec{v}_k$ (not $\vec{v}_1 \cdot \vec{v}_k$) and the last entry should be $|\vec{v}_k|^2$ (not $|\vec{v}_n|^2$). In the third printing we will put some of the other entries in a different order, so that they are consistent, with the vector coming from T^\top coming first. Thus the matrix will be

$$\begin{bmatrix} |\vec{v}_1|^2 & \vec{v}_1 \cdot \vec{v}_2 & \dots & \vec{v}_1 \cdot \vec{v}_k \\ \vec{v}_2 \cdot \vec{v}_1 & |\vec{v}_2|^2 & \dots & \vec{v}_2 \cdot \vec{v}_k \\ \vdots & \vdots & \ddots & \vdots \\ \vec{v}_k \cdot \vec{v}_1 & \vec{v}_k \cdot \vec{v}_2 & \dots & |\vec{v}_k|^2 \end{bmatrix}$$

p. 472 The matrix in Equation 5.1.11 should have the same corrections as the same matrix in Equation 5.1.3; in particular, the subscript n 's should be k 's, and the \vec{v}_1 in the entry $\vec{v}_1 \cdot \vec{v}_n$ should be \vec{v}_2 .

p.473 Third line of Section 5.2: “mapping γ ” not “mapping φ .” In keeping with the above correction for page 272, it should be “ $U \subset \mathbb{R}^k$ ” not “ $U \subset \mathbb{R}^n$.”

p. 474, Definition 5.2.1. should say “bounded subset $X \subset \mathbb{R}^n$ ” and “arbitrary subset $X \subset \mathbb{R}^n$.”

p. 474 Clarification, not correction: in Equation 5.2.2 we are summing over the sidelength of C , to the k :

$$\lim_{N \rightarrow \infty} \sum_{\substack{C \in \mathcal{D}_N(\mathbb{R}^n) \\ C \cap X \neq \emptyset}} \left(\underbrace{\frac{1}{2^N}}_{\substack{\text{sidelength} \\ \text{of } C}} \right)^k = 0.$$

p. 474, Definition 5.2.2, in (3), there should be a colon: $\gamma : (U - X) \rightarrow M$, not $\gamma(U - X) \rightarrow M$. The caption to Figure 5.2.1 should be “The subset of \mathbb{R}^3 of equation $x^2 + y^2 = z^2$ is not a manifold at the vertex.”

p. 476 Equation 5.2.12 should be

$$t \mapsto \begin{pmatrix} x = 1 - t^2 \\ z = t^3 \end{pmatrix}.$$

In addition, we have added a margin note for Example 5.2.5:

If you take any equation representing a curve in the (x, z) -plane and replace x by $\sqrt{x^2 + y^2}$, you get the equation of the surface obtained by rotating the original curve around the x -axis. The surface is symmetric about the z -axis; if $y = 0$, the two equations are identical.

p. 477 Immediately after Equation 5.2.13, “Figure 5.2.3 represents the image of the parametrization” should be

“Figure 5.2.3 represents the image of this parametrization for” . . .

p. 477 First line after the subheading, it would be better to say “entire theory of integrals over manifolds.” Theorem 5.2.6: the manifold M is k -dimensional.

p. 477, the sidebox concerning Exercise 5.2.6 is incorrect; the manifold in that exercise *can* be parametrized.

p. 479, In the first margin note, just before the displayed equation, φ should be Φ , and D should be \mathbf{D} : “then

$$(f \circ \Phi) |\det[\mathbf{D}\Phi]|$$

is also integrable.”

p. 481, Proposition 5.3.3, “same curve $C \subset \mathbb{R}^3$ ” not $\in \mathbb{R}^3$.

p. 482 Two lines before Equation 5.3.18, T_1 should be t_1 .

p. 482 Although Equation 5.3.18 is correct as stated, we prefer it written as a matrix multiplication (no \circ on the right-hand side):

$$[\mathbf{D}(\gamma_2 \circ \Phi)(t_1)] = [\mathbf{D}\gamma_2(\Phi(t_1))][\mathbf{D}\Phi(t_1)].$$

p. 482 In Equation 5.3.23, first line, the first term on the right is wrong. The equation should be

$$\begin{aligned} \int_{I_2} |\vec{\gamma}_2'(t_2)| |dt_2| &= \int_{I_1} |[\mathbf{D}\gamma_2(\Phi(t_1))]| \cdot \overbrace{|\mathbf{D}\Phi(t_1)|}^{\det[\mathbf{D}\Phi(t_1)]} |dt_1| \\ &= \int_{I_1} |[\mathbf{D}\gamma_1(t_1)]| |dt_1| = \int_{I_1} |\vec{\gamma}_1'(t_1)| |dt_1|. \quad \square \end{aligned}$$

p. 484 In Examples 5.4.2 and 5.4.3 we compute the area using the length of the cross-product; when we switched to using one formula ($\sqrt{\det T^\top T}$) for all cases, we forgot to change the examples. Of course for surface area, using the cross product is correct (see Proposition 1.4.19), but we think it easier to stick with $\sqrt{\det T^\top T}$.

p. 486 The second line of Equation 5.4.16 is incorrect; the equation should read

$$\begin{aligned} &\int_V \sqrt{\det[\mathbf{D}\gamma_2(\mathbf{v})]^\top [\mathbf{D}\gamma_2(\mathbf{v})]} |d^2\mathbf{v}| \\ &= \int_U \sqrt{\det([\mathbf{D}\gamma_2(\Phi(\mathbf{u}))]^\top [\mathbf{D}\gamma_2(\Phi(\mathbf{u}))])} |\det[\mathbf{D}\Phi(\mathbf{u})]| |d^2\mathbf{u}| \\ &= \int_U \sqrt{\det([\mathbf{D}\gamma_2(\Phi(\mathbf{u}))]^\top [\mathbf{D}\gamma_2(\Phi(\mathbf{u}))])} \sqrt{\det([\mathbf{D}\Phi(\mathbf{u})]^\top [\mathbf{D}\Phi(\mathbf{u})])} |d^2\mathbf{u}| \\ &= \int_U \sqrt{\det([\mathbf{D}\Phi(\mathbf{u})]^\top [\mathbf{D}\gamma_2(\Phi(\mathbf{u}))]^\top [\mathbf{D}\gamma_2(\Phi(\mathbf{u}))][\mathbf{D}\Phi(\mathbf{u})])} |d^2\mathbf{u}| \\ &= \int_U \sqrt{\det([\mathbf{D}(\gamma_2 \circ \Phi)(\mathbf{u})]^\top [\mathbf{D}(\gamma_2 \circ \Phi)(\mathbf{u})])} |d^2\mathbf{u}| \\ &= \int_U \sqrt{\det[\mathbf{D}\gamma_1(\mathbf{u})]^\top [\mathbf{D}\gamma_1(\mathbf{u})]} |d^2\mathbf{u}|. \quad \square \end{aligned}$$

p. 487 in Equation 5.4.19, the last entry on the right should be $r_2 \sin v$, not $r_1 \sin v$.

p. 487: In the sentence immediately after Equation 5.4.20, referring to Equation 5.4.3 is inadequate justification. In this example we are using Theorem 5.1.4 giving the volume of a k -parallelogram in \mathbb{R}^n , combined with Proposition 5.5.1, which says that the volume of a manifold is independent of the parametrization used. Thus Proposition 5.4.4 concerning surfaces in \mathbb{R}^3 is true for surfaces in \mathbb{R}^n .

p. 488 In Equation 5.4.25, second line, second matrix: the last entry of the first column should be $2r \sin 2\theta$, not $2r \sin \theta$. In the second line of Equation 5.4.26, $|du dv|$ should be $|dr d\theta|$.

p. 488 In the second paragraph of Section 5.5, the sentence beginning “That is, we find a subset...” should be deleted, as it refers to the original strict definition of parametrizations, not the relaxed definition (Definition 5.2.2) that we are now using.

p. 489 Proposition 5.5.1 should read: “Let M be a k -dimensional manifold in \mathbb{R}^n . If U and V are subsets of \mathbb{R}^k and $\gamma_1 : U \rightarrow M$, $\gamma_2 : V \rightarrow M$ are two parametrizations of M , then ...”

p. 489 In the second line of Equation 5.5.3, first matrix, the entry of the third column, third row should be $D_3 f$, not $D_2 f$.

p. 490 In the heading of the table, s_n , which was never defined properly, should be replaced by $\text{vol}_n S^n$. In addition, \log has been changed to \ln

p. 491 \log has been changed to \ln . There is an ambiguous exponent in Equation 5.6.2; the equation is now

$$4^n \left(\frac{1}{3}\right)^{n \ln 4 / \ln 3} = 4^n e^{n \frac{\ln 4}{\ln 3} (\ln \frac{1}{3})} = 4^n e^{n \frac{\ln 4}{\ln 3} (-\ln 3)} = 4^n e^{-n \ln 4} = \frac{4^n}{4^n} = 1.$$

p. 492 Exercise 5.1.3: “Show that if \vec{v}_1, \vec{v}_k are linearly dependent” should be “Show that if $\vec{v}_1, \dots, \vec{v}_k$ are linearly dependent.”

p. 493, Exercise 5.2.5 now reads

(a) Show that the graph of a C^1 mapping from $\mathbb{R}^l \rightarrow \mathbb{R}^{n-l}$ for $l < k$ has k -dimensional volume 0.

(b) Show that if $M \subset \mathbb{R}^n$ is a closed l -dimensional manifold with $l < k$, then $\text{vol}_k(M) = 0$.

The proof uses the Heine-Borel theorem (Theorem A17.2).

p. 493, Exercise 5.2.6 is wrong. It should read “... and that it can be parametrized,” not “... and that it cannot be parametrized.”

p. 496 In Exercise 5.4.13, “triangulation of the unit square” means “decomposing the unit square into triangles.” In Exercise 5.5.7, $-\pi/2 \leq \varphi, \psi \leq \pi/2, 0 \leq \theta < 2\pi$ might be clearer written $-\pi/2 \leq \varphi \leq \pi/2$ and $-\pi/2 \leq \psi \leq \pi/2, 0 \leq \theta < 2\pi$.

p. 497 In Exercise 5.6.2, the fifths are all open.

In Exercise 5.6.3, we should have specified that the intervals that are removed are open. The “middle fifth” should be “middle $1/n$ th.” If n is even, it might be better to say “remove $1/n$ th of the unit interval, leaving equal amounts on both sides.”