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The KAM Theorem

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I first heard about the KAM theorem when I was an undergraduate, around 1966. It seemed to me the most beautiful result in the world, but for many years my interests were engaged elsewhere. Around 1980, I came back to dynamical systems, and I quickly realized that the KAM theorem is indispensable.

Each year, for about fifteen years, I said to myself in September: this is the year that I am going to understand the proof. Each year, as March came around, I had to admit failure once again: I no longer knew the order of the quantifiers in the technical lemmas, and so was unable to apply them.

During these years, I tackled all the proofs that I knew: Arnold's [Arn63, AA68], Moser's [Mos62, Mos73], Sternberg's [Ste71], those based on the Nash–Hamilton implicit function theorem, those of Herman [FH83, Her86],... I did not succeed in mastering a single one. And I am far from being alone: I know numerous dynamicists who realize that they ought be able to prove the theorem, who even teach it sometimes, but who have never mastered the proof either.

After being pointed in the right direction by Pierre Lochak, I finally discovered the article of Bennettin, Galgani, Giorgilli and Strelcyn [BGGS84], which I found luminous. With the help of Yulij Ilyashenko, I discovered several improvements: this is the proof published in [HI02]. Ilyashenko gave an exposition of it at the Moscow mathematics seminar in 2002; in the audience were some participants from Kolmogorov's seminar in 1957; they told him that this proof was in fact the original proof.

One might wonder whether this is really true. At any rate, it is very hard to understand why Kolmogorov never published a proof of his most beautiful result.