## New Errata: Nov. 16, 2000

Following are all errors since the June 2,2000 posting. Page numbers refer to the third printing; in the vast majority of cases they should correspond to page numbers in the first and second printings. (We are currently in France and don't have copies of the first and second printings.)

**p. 38** Example 1.2.6: the second line should be "product of A and the second column of B."

**p. 98** Last line: "for small u" should be "for small |u|." (also in underbrace in Equation 1.6.25).

**p. 99** Two lines after Equation 1.6.26: sentence beginning "If we were to forget about the small terms grouped in parentheses on the right-hand side of ... " should read

"If we were to forget about the small terms grouped in parentheses on the right-hand side of Equation 1.6.25, we would say that these points travel in a circle of radius  $|b_j|\rho^j$  around the point  $b_0 = p(z_0)$ ."

**p. 110** The margin note that begins "Once we know the partial derivatives ... " should read "If **f** is differentiable, then once we know the partial derivatives of **f**, which measure rate of change

in the direction of the standard basis vectors, we can compute the derivatives in *any* direction." (If **f** is not differentiable, the extrapolation does not work.)

**p. 138** Exercise 1.5.18 belongs in Section 1.6

**p. 191** The statement in the second margin note, that "in infinite-dimensional vector spaces, bases usually do not exist," should be, "In infinite-dimensional vector spaces, bases usually cannot be constructed."

**p. 193** Definition 2.6.11: "An ordered set of vectors," not "a set of vectors." Definition 2.6.12: "a finite, ordered collection" not "a finite collection."

**p. 225** Immediately after Equation 2.9.15: The sentence after "We want to find the image of F" should read

"If at a point  $\begin{pmatrix} \theta \\ \varphi \end{pmatrix}$  the derivative of F is invertible,  $F\begin{pmatrix} \theta \\ \varphi \end{pmatrix}$  will certainly be in the *interior* of the image, since points in the neighborhood of that point are also in the image."

(To understand the image, we need to know where it ends: where its boundary is. So we will look first for the boundary of the image, then determine what locus can have that boundary.)

**p. 262** Proposition 3.1.19, part (b): C not X.

**p. 267** Last line of p. 267, continuing to p. 268: The sentence "It is true that for a given  $\mathbf{x}_1$  and  $\mathbf{x}_3$ , four positions are possible in all, but ... " should be

"If for a given  $\mathbf{x}_1$  and  $\mathbf{x}_3$  four positions are possible. If the rods are in one of these positions, and you move  $\mathbf{x}_1$  and  $\mathbf{x}_3$  a *small* amount, then there will be only one possible position of the linkage, and hence of  $\mathbf{x}_2$  and  $\mathbf{x}_4$ , near the starting position."

**p. 274** Line immediately after Equation 3.2.8: "which is simply saying  $[\mathbf{Df}(\vec{\mathbf{v}} + \vec{\mathbf{w}})] = 0$ " should be

"which is simply saying  $[\mathbf{Df}(\mathbf{x})][\vec{\mathbf{v}} + \vec{\mathbf{w}}] = 0$ ."

**p. 285** We have replaced the proof of Theorem 3.3.18 by a shorter proof:

(a) Consider the polynomial  $Q_{f,\mathbf{a}}^k$  that, evaluated at  $\mathbf{\vec{h}}$ , gives the same result as the Taylor polynomial  $P_{f,\mathbf{a}}^k$  evaluated at  $\mathbf{a} + \mathbf{\vec{h}}$ :

multi-exponent notation for polynomial  $Q_{f,\mathbf{a}}^k$ 

$$P_{f,\mathbf{a}}^{k}(\mathbf{a}+\vec{\mathbf{h}}) = Q_{f,\mathbf{a}}^{k}(\vec{\mathbf{h}}) = \sum_{m=0}^{k} \sum_{J \in \mathcal{I}_{n}^{m}} \frac{1}{J!} D_{J}f(\mathbf{a}) \underbrace{\vec{\mathbf{h}}_{J}^{J}}_{\text{variable}}$$

By Proposition 3.3.12,

$$\underbrace{D_I Q_{f,\mathbf{a}}^k(\mathbf{0})}_{D_I p(\mathbf{0})} = I! \underbrace{\frac{1}{I!} D_I f(\mathbf{a})}_{a_I} = D_I f(\mathbf{a}).$$

When  $\vec{\mathbf{h}} = \mathbf{0}$ , then  $P_{f,\mathbf{a}}^k(\mathbf{a} + \vec{\mathbf{h}})$  becomes  $P_{f,\mathbf{a}}^k(\mathbf{a})$ , so

$$D_I Q_{f,\mathbf{a}}^k(\mathbf{0}) = D_I P_{f,\mathbf{a}}^k(\mathbf{a}), \text{ so } D_I P_{f,\mathbf{a}}^k(\mathbf{a}) = D_I f(\mathbf{a});$$

the partial derivatives of  $P_{f,\mathbf{a}}^k$ , up to order k, are the same as the partial derivatives of f, up to order k.

Part (b) then follows from Proposition 3.3.19. To lighten the notation, denote by  $g(\mathbf{a} + \mathbf{h})$  the difference between  $f(\mathbf{a} + \mathbf{h})$  and the Taylor polynomial of f at  $\mathbf{a}$ . Since all the partials of g up to order k vanish, Proposition 3.3.19 says that

$$\lim_{\vec{\mathbf{h}}\to\mathbf{0}}\frac{g(\mathbf{a}+\vec{\mathbf{h}})}{|\vec{\mathbf{h}}|^k}=0. \quad \Box$$

p. 301 Theorem 9.8.5 should be Theorem 3.6.8, here and on next page

**p.** 307 Third line from the bottom, and in the caption to Figure 3.7.4: each root is the first coordinate of a point whose image under  $\mathbf{g}$  is a critical point of f constrained to X (not the first coordinate of a critical point).

**p. 315** In Equation 3.7.37,  $\vec{\mathbf{v}}_1$  should be  $\vec{\mathbf{v}}_1^{\top}$ . In Equation 3.7.38,  $\mu_{2,1}\vec{\mathbf{v}}_1$  should be  $\frac{\mu_{2,1}}{2}\vec{\mathbf{v}}_1$ . Similarly,  $\mu_{2,1}$  should be divided by 2 in Equation 3.7.40

p. 360 The second margin note, last sentence: Proposition 4.1.12, not 4.1.37.

**p. 385** Next-to-last paragraph: x and y should be  $\mathbf{x}$  and  $\mathbf{y}$ 

**p. 386** Equation 4.4.8: x + a, not x - a

**p. 407** Sentence immediately after Equation 4.8.8: "how to compute larger ones" should be "how to compute determinants of larger ones."

p. 419 Sentence immediately after Equation 4.8.55, "If a term" should be should be "if a term."

**p. 471** Last line: k = 3 not n = 3

**p. 472** Proof of Theorem 5.1.4:  $k \times k$  matrix, not  $n \times n$ 

**p. 476** Equations 5.2.10 and 5.2.11: to be consistent,  $\phi$  should be  $\varphi$ 

**p. 503** In the paragraph beginning, "We can now interpret...": "the  $(x_1, x_2, x_4)$  component of signed volume" not "the  $dx_1 dx_2 d_4$  component".

**p. 534** Last margin note, last sentence: this matrix is the derivative of the "change of parameters" map, not this matrix is the "change of parameters" map.