

Corrections to the Second Printing: Chapters 0 and 1

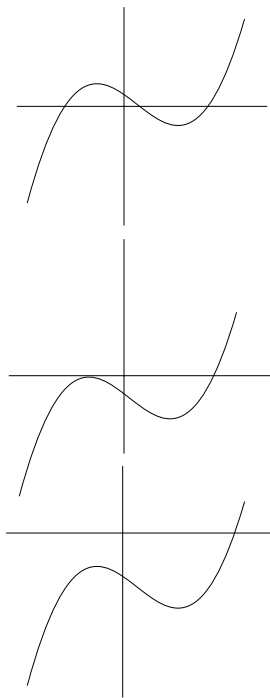
Corrections to Chapter 0

p. 9, Equation 0.4.3, $\sup$ and $\inf$ are undefined; see Definitions 1.6.4 and 1.6.6.

p 11 since Theorem 0.4.10 is an “if and only if” statement, the proof should deal with both directions; thus it should start, “If a sequence a_n converges, it is clearly bounded. If it is bounded, it has a least upper bound A .”

p. 17 Part (3) of the proof of Proposition 0.6.5: “do not differ by an integer multiple of 2π ,” not “by a multiple of 2π .”

p.20 Figure 0.6.3 does not correspond to the caption. The figure should be



p. 21 Exercise 0.4.3: we think this will be clearer if the last sentence reads, “ Notice that for A and S , the l of Definition 0.4.4 does not depend on N , but that for M , l does depend on N .”

Corrections to Chapter One

p. 33, line 1: “A non-empty subset $V \subset \mathbb{R}^n$ ” not “A non-empty subset $V \in \mathbb{R}^n$.”

p. 39 Two lines before Proposition 1.2.8, “Is $(AB)C$ the same as $(AB)C?$ ” should be “Is $(AB)C$ the same as $A(BC)?$ ”.

p. 46 The top of the page might be clearer with a few changes:

First notice that $B_1 = A^1 = A$: the entry $A_{i,j}$ of the matrix A is exactly the number of walks of length 1 from v_i to v_j .

Next, suppose it is true for n , and let us see that it is true for $n + 1$. A walk of length $n + 1$ from V_i to V_j must be at some vertex V_k at time n . The number of such walks is the sum, over all such V_k , of the number of ways of getting from V_i to V_k in n steps, times the number of ways of getting from V_k to V_j in one step (which will be 1 if V_k is next to V_j , and 0 otherwise).

p. 64, Second margin note: the mention of “Minkowski norm” appears to be incorrect. What we call the length of a matrix is called the Frobenius norm, the Schur norm and Hilbert-Schmidt norm.

p. 66, proof of Proposition 1.4.14: we should have said that θ is the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$.

p. 77 Add commas two lines before Equation 1.5.10, and in Equation 1.5.11: $M_1, \dots, M_n$.

p. 81 In Definition 1.5.19, we began with a mixture of f ’s and $\mathbf{f}$ ’s and then “corrected” the f ’s to $\mathbf{f}$ ’s. That was wrong. Here we are discussing a scalar-valued function $f : U \rightarrow \mathbb{R}$, not an $\mathbb{R}^m$ -valued function $\mathbf{f} : U \rightarrow \mathbb{R}^m$. In the definition and following text (through Equation 1.5.18), the $\mathbf{f}$ ’s should all be f ’s, and the $\mathbf{a}$ ’s should be a ’s.

p. 82 The line immediately before Equation 1.5.22 should read “If $|\mathbf{x} - \mathbf{x}_0| < \delta$, then $|\mathbf{x} - \mathbf{x}_0| < \delta_i$, so that $|f_i(\mathbf{x}) - a_i| < \epsilon$, so that

p. 86 First sentence: replace “There is a reformulation in terms of epsilons and deltas” by

“The following criterion shows that it is enough to consider $\mathbf{f}$ on sequences converging to $\mathbf{x}_0$.”

Replace Proposition 1.5.26 by

Proposition 1.5.26 (Criterion for continuity). *The map $\mathbf{f} : X \rightarrow \mathbb{R}^m$ is continuous at $\mathbf{x}_0$ if and only if for every sequence $\mathbf{x}_i \in X$ converging to $\mathbf{x}_0$,*

$$\lim_{i \rightarrow \infty} \mathbf{f}(\mathbf{x}_i) = \mathbf{f}(\mathbf{x}_0).$$

p. 86 Theorem 1.5.27 part (e) should read:

If h is continuous at $\mathbf{x}_0$, with $h(\mathbf{x}_0) = 0$, and $\mathbf{f}$ is bounded in a neighborhood of $\mathbf{x}_0$, then $h\mathbf{f}$ is continuous at $\mathbf{x}_0$ (even if $\mathbf{f}$ is not defined at $\mathbf{x}_0$).

p. 90 Three lines above Equation 1.6.1, “a second box $B_1 \subset B_0$ of side $1/10$,” not “a second box $B_1 \in B_0$ of side $1/10$.”

p. 98, Equation 1.6.22 is wrong: it should be

$$(r(\cos \theta + i \sin \theta))^k = r^k(\cos k\theta + i \sin k\theta).$$

(This equation was stated correctly in Corollary 0.6.4)

p. 101, right after Definition 1.7.1, “about open sets $U \subset \mathbb{R}$ ” not “about open sets $U \in \mathbb{R}$.”

p. 106 There should be parentheses in the right-hand sides of Equations 1.7.18 and 1.7.19:

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{1}{h} \left((f(a+h) - f(a)) - [f'(a)]h \right) &= \lim_{h \rightarrow 0} \left(\frac{f(a+h) - f(a)}{h} - \frac{f'(a)h}{h} \right) \\ &= f'(a) - f'(a) = 0. \end{aligned} \quad 1.7.18$$

and

$$0 = \lim_{h \rightarrow 0} \frac{1}{h} \left((f(a+h) - f(a)) - mh \right) = \lim_{h \rightarrow 0} \left(\frac{f(a+h) - f(a)}{h} - \frac{mh}{h} \right) = f'(a) - m, \quad 1.7.19$$

p. 107 Definition 1.7.8 should specify the dimensions of $\mathbf{f}$: “The Jacobian matrix of a function $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is the $m \times n$ matrix”

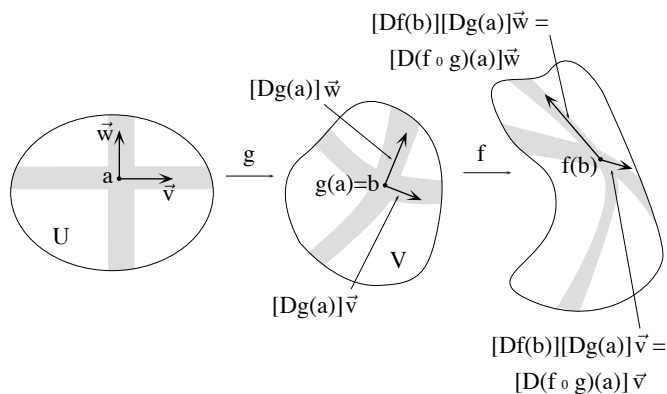
p. 111 Margin note: “...gives $f \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and we are asking...” not “gives $f \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.”

p. 116 In Equation 1.8.4, it might be clearer to write $[\mathbf{D}\mathbf{f}(\mathbf{a})]\vec{\mathbf{v}}$ rather than $f'(\mathbf{a})\vec{\mathbf{v}}$. In the last sentence of the first margin note, $[\mathbf{D}\mathbf{g}(\mathbf{a})]\vec{\mathbf{v}}$ should be replaced by $[\mathbf{D}\mathbf{f}(\mathbf{a})]\vec{\mathbf{v}}$

p. 117 In part (4) of the proof, f and g should be $\mathbf{f}$ and $\mathbf{g}$.

p. 117, In Equation 1.8.8, $\frac{1}{\mathbf{h}}$ should be $\frac{1}{|\mathbf{h}|}$ in three places (one on each line).

p. 118 There are errors in the labels of figure 1.8.1 and in the caption. It should be



Caption: The function $\mathbf{g}$ maps a point $\mathbf{a} \in U$ to a point $\mathbf{g}(\mathbf{a}) \in V$. The function $\mathbf{f}$ maps the point $\mathbf{g}(\mathbf{a}) = \mathbf{b}$ to the point $\mathbf{f}(\mathbf{b})$. The derivative of $\mathbf{g}$ maps the vector $\vec{\mathbf{v}}$ to $[\mathbf{D}\mathbf{g}(\mathbf{a})](\vec{\mathbf{v}})$. The derivative of $\mathbf{f} \circ \mathbf{g}$ maps $\vec{\mathbf{v}}$ to $[\mathbf{D}\mathbf{f}(\mathbf{b})][\mathbf{D}\mathbf{g}(\mathbf{a})]\vec{\mathbf{v}}$, i.e., to $[\mathbf{D}\mathbf{f}(\mathbf{g}(\mathbf{a}))][\mathbf{D}\mathbf{g}(\mathbf{a})]\vec{\mathbf{v}}$.

p. 121 Equation 1.9.5 is missing an “as $s \rightarrow 0$ ”; it should be

$$g'(t_0) = \lim_{s \rightarrow 0} \frac{g(t_0 + s) - g(t_0)}{s} = \lim_{s \rightarrow 0} \frac{f(\mathbf{c} + s(\mathbf{b} - \mathbf{a})) - f(\mathbf{c})}{s} = [\mathbf{D}f(\mathbf{c})](\mathbf{b} - \mathbf{a}).$$

p. 130 Exercise 1.2.15 (e) should read

Show that you can color the vertices of a connected graph (one on which you can walk from any vertex to any other) in two colors, such that no two adjacent vertices have the same color, if and

only if for all sufficiently high powers n of the adjacency matrix, those entries that are 0 for A^n are nonzero for A^{n+1} , and those that are nonzero for A^n are zero for A^{n+1} .

p. 131 Some students found the wording of Exercise 1.2.22 confusing. It should read

What 2×2 matrices A satisfy

$$(a) A^2 = 0, \quad (b) A^2 = I, \quad (c) A^2 = -I?$$

p. 132 Exercise 1.3.11, each homework counts for 2.5 percent of the final grade, not 1.5 percent, and we should have asked, “What matrix operation should one perform to assign to each student his or her final grade?”

p.134, Exercise 1.4.8 is the same as 1.4.4(c)

p. 136, Exercise 1.5.4, the first part of part (c) should have e^A , not a^A :

Do you think that $e^{A+B} = e^A e^B$ for all A and B ?

p. 138, Exercise 1.5.15 (b) should be:

For the two functions below, defined on $\mathbb{R}^2 - \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$, either show that the limit exists at $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and find it, or show that it does not exist.

p. 142 Exercise 1.7.14 like Exercise 1.7.13 part b.

p. 143 Exercise 1.7.16 part (a) should be with the exercises for Section 1.8.

p.144 Exercise 1.8.4 differentiable, not smooth

p. 145 Exercise 1.8.9: $[D\mathbf{f}(\mathbf{0})]$ should be $[\mathbf{Df}(\mathbf{0})]$ and $\mathbf{g} \circ \mathbf{f}(\mathbf{x})$ should be $(\mathbf{g} \circ \mathbf{f})(\mathbf{x})$.

p. 146 The second margin note: $|\sin x| \leq |x|$ not $|\sin x| \leq x$