Corrections to the Second Printing: Chapter 4

p. 358, the last margin note should read:

Since we are in dimension 1, our cubes are intervals:

$$C_{k,N} = \left[\frac{k}{2^N}, \frac{k+1}{2^N}\right).$$

p. 360, the proof of Proposition 4.1.12 should begin with the sentence, "We will prove this in the case where f_1 and f_2 are nonnegative." A margin note should read, "The general case of Proposition 4.1.12 can be proved using $f_1 = f_1^+ + f_1^-$, $f_2 = f_2^+ + f_2^-$ (see Definition 4.3.4)." When time permits, this proof will be added to the web page.

p. 368, Equation 4.2.13, the integrals are over $\mathbb{R}$, not R.

p. 370, Equation 4.2.18: the exponent needs a minus sign. The normal distribution is given by

$$\mu(x) = \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}.$$

p. 370 In Theorem 4.2.11, "expectation value" should be "expectation" (or "expected value").

p. 371 Equation 4.2.23 in the numerator, $\sqrt{1000}$, not $2\sqrt{1000}$

p. 372 Second line: within two standard deviations of the mean, not one; as a result, we want $\frac{1}{2}\%$ to be the standard deviation, and Equation 4.2.25 should be

$$\frac{1}{2\sqrt{n}} = \frac{1}{200}; \quad \text{i.e.,} \quad n = 10000.$$

The answer in the footnote should be 2500, not 625.

p. 377 In the second margin note, it should be " ... we will be able to drop the requirement that X be compact," not "that M be compact."

p. 380, Equation 4.3.18 should be

$$y = +\sqrt{1-x^2} \quad \text{and} \quad y = -\sqrt{1-x^2}.$$

p. 381, in Definition 4.4.1, "$X \subset \mathbb{R}^n$," not $X \in \mathbb{R}^n$, and "$X \subset \cup B_i$," not $X \in \cup B_i$."

p. 383 The caption to Figure 4.4.2 should read: The trajectory with slope 2/5, at center, visits more of the square than the trajectory with slope 1/2, at left. The slope of the trajectory at right closely approximates an irrational number; if allowed to continue, this trajectory would visit nearly every part of the square. (In practice, the drawing would soon become black.)

p. 384 Equation 4.4.4 should have $\subset$, not $\in$: $\Delta \subset \cup B_i$.

p. 394, The caption to Figure 4.5.10 should read: The situation if the first dart lands above the diagonal. If the second dart lands in the shaded area, then the determinant is negative because the second vector is clockwise from the first (Proposition 1.4.14). For values of x_2 to the right of the vertical dotted line (which has x-coordinate x_1), y_2 goes from 0 to 1. For values of x_2 to the left of the vertical dotted line, we must consider separately values of y_2 from 0 to $\frac{y_1 x_2}{x_1}$ (the shaded region, with $-\det$) and values of y_2 from $\frac{y_1 x_2}{x_1}$ to 1 (the unshaded region, with $+\det$).

p. 394 Strictly speaking, when we compute the first term of the inner integral, it should be

$$-\overbrace{0}^{\text{eval. at } y_2=0} \qquad \text{not } +0.$$

p. 394, footnote 7 is wrong. It should read:

Exercise 4.5.7 asks you to compute the integral that way. The integral is then the sum of the integral for the half of the square below the diagonal (given by Equation 4.5.25), and the integral for the half above the diagonal. The latter is

$$\int_0^1 \int_{x_1}^1 \int_0^{\frac{x_1}{y_1}} \overbrace{\left(\int_0^{\frac{y_1 x_2}{x_1}} \underbrace{(x_2 y_1 - x_1 y_2)}_{-\det} \, dy_2 + \int_{\frac{y_1 x_2}{x_1}}^1 \underbrace{(x_1 y_2 - x_2 y_1)}_{+\det} \, dy_2 \right)}^{\text{for } x_2 \text{ to the left of the vertical line with } x\text{-coordinate } x_1} dx_2 \, dy_1 \, dx_1$$
$$+ \quad \int_0^1 \int_{x_1}^1 \int_{\frac{x_1}{y_1}}^1 \underbrace{\left(\int_0^1 (x_2 y_1 - x_1 y_2) \, dy_2 \right)}_{\substack{\text{for } x_2 \text{ to the right of the vertical line} \\ \text{with } x\text{-coordinate } x_1;\ -\det}} dx_2 \, dy_1 \, dx_1.$$

p. 395 In the underbrace for the third line of Equation 4.5.27, $y_1^2 4$ should be $\frac{y_1^2}{4}$.

p. 399 In $\int_0^\infty f(x) e^{-x} \, dx$, the function $f(x)$ is weighted by e^{-x}; when we speak of "functions with a particular weight" we are not referring to the weights of Guassian rules or Simpson's method.

p. 401 In the margin note, it should be $w_1 = w_3$, not $w_1 = w_2$; i.e., the one-dimensional weights are

$$w_1 = w_3 = \frac{b-a}{6n} \cdot 1 = \frac{1}{3} \quad \text{and} \quad w_2 = \frac{1}{3}(4) = \frac{4}{3}.$$

p. 402 In Definition 4.6.7, part (3)

$$\overline{s}^2 = \frac{1}{N} \sum_{i=1}^N a_i - \overline{a}^2 \quad \text{should be} \quad \overline{s}^2 = \frac{1}{N} \sum_{i=1}^N (a_i - \overline{a})^2$$

p. 403 In the bottom margin note, $\mathbf{x}_9$ should be x_9 (not bold).

p. 404 Definition 4.7.3 has been changed. As written, the boundary of a paving does not include the border between a tile P and $\mathbb{R}^n - X$. We have added a definition of "boundary" to Section 1.5:

The boundary of a subset $A \subset \mathbb{R}^n$ is those points for which every neighborhood intersects both A and the complement of A. It is also the intersection of the closure of A and the closure of the complement of A.

and have changed Definition 4.7.3 to read

The boundary $\partial \mathcal{P}$ of $\mathcal{P}$ is the union of the boundaries of the tiles:

$$\partial \mathcal{P} = \bigcup_{P \in \mathcal{P}} \partial P.$$

p. 404 last margin note, "The set of all $P \in \mathcal{P}$ completely paves $X \subset \mathbb{R}^n$," not "The set of all $P \in \mathcal{P}$ completely paves $\mathbb{R}^n$."

p. 408, clarification, not correction: top of the page: we define $A_{i,j}$ but in Equation 4.8.9 we have the case where $j = 1$: $A_{i,1}$, the $(n-1) \times (n-1)$ matrix obtained from A by erasing the ith row and the first column.

p. 409 next to last line: Section 2.3, not Section 2.1

p. 413, third line: it would be better to replace "so $\det E = \det I$" by "so $\det E = 1$." Fifth line: replace "$\det E^\top = \det I$" by "$\det E^\top = 1$."

p. 414 Since we discuss the change of basis matrix only in volume two, it would be better not to mention it in Theorem 4.8.13. That theorem, and the preceding paragraph, should be:

The following theorem acquires its real significance in the context of abstract vector spaces, where we will see that it means that the determinant function is basis independent. This is discussed in volume two; in this volume we will find the theorem useful in proving Corollary 4.8.22.

Theorem 4.8.13. *If P is invertible, then*

$$\det A = \det(P^{-1}AP).$$

p. 416 Theorem 4.8.18: it is probably obvious, but we should have said that $\text{Perm}(1, \dots, n)$ denotes the set of permutations of $1, \dots, n$.

p. 417 Right after the table, "So $\det A = 45 + 84 + 96 - 48 - 72 - 105 = 0$" ($-48$, not -42).

p. 419 In the proof of part (b), the word "term" may not be clear. The beginning of that proof should read:

... Put another way, we want to find the terms that are linear in h of the expansion given by Equation 4.8.46 for

$$\det(I + hB) = \det \begin{bmatrix} 1 + hb_{1,1} & hb_{1,2} & \dots & hb_{1,n} \\ hb_{2,1} & 1 + hb_{2,2} & \dots & hb_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ hb_{n,1} & hb_{n,2} & \dots & 1 + hb_{n,n} \end{bmatrix}.$$

Equation 4.8.56 shows ... .

p. 419 In the bottom margin note, the matrix in the second line of the equation should be preceded by det. In Equation 4.8.57, if you have trouble getting from the next-to-last line to the last line, replace the 1 in the next-to-last line by $\det I$; the limit in that line is then the directional derivative of det at I in the direction $A^{-1}B$, which by Proposition 1.7.12 can be written $[\mathbf{D}\det(I)](A^{-1}B)$, which by part (b) of the theorem is $\operatorname{tr}(A^{-1}B)$.

p. 421 The last margin note should say "The first requirement for a paving of $\mathbb{R}^n$," not "The first requirement for a paving."

p. 426 Although it makes for a lot of parentheses, the sum in the last line of Equation 4.9.19 should perhaps be written

$$\sum_{P\in(T(\mathcal{D}_N(\mathbb{R}^n)))};$$

we are summing over those P that are in the decomposition $T(\mathcal{D}_N(\mathbb{R}^n))$.

p. 426, Definition 4.9.8: "parallelepiped" should be "k-parallelogram."

p. 427 The margin note should be changed, since this issue is addressed only in the sequel to the present volume. It should read:

The meaning of expressions like du is explored in Chapter 6. In volume two we will see that we can use the change of variables formula in higher dimensions without requiring exact correspondence of domains, using the language of forms. This is what you were using (more or less blindly) in one dimension.

p. 430, caption to Figure 4.10.4: φ is latitude, not longitude.

p. 431 The third margin note is wrong. It should be "At $\varphi = -\pi/2$ and $\varphi = \pi/2$, $r = 0$." Two lines before Definition 4.10.9, "projection in" should be "projection onto." In the third line of Definition 4.10.9, "the projection" should be "its projection."

p. 432 The last margin note should be deleted; there is no V in the theorem. We have changed names of variables in in Equation 4.10.21; it now reads

$$\int_Y f(\mathbf{y})\,|d^n\mathbf{y}| = \int_X (f\circ\Phi)(\mathbf{x})\,\big|\det[\mathbf{D}\Phi(\mathbf{x})]\big|\,|d^n\mathbf{x}|.$$

p. 433 Three lines before Equation 4.10.4, "since $\det[DP] = r$" not "since $\det[D\Phi] = r$."

p. 440 Proposition 4.11.7, $M \subset \mathbb{R}^n$ not $M \in \mathbb{R}^n$. The title may be misleading; we have changed it and part (a) to read

Proposition 4.11.7 (Manifold of lower dimension has volume 0) (a) Any manifold $M \subset \mathbb{R}^n$ that is a closed subset of $\mathbb{R}^n$ and of dimension less than n has n-dimensional volume 0.

p. 441 First margin note: K, not N, to be consistent with the notation of Definition 4.11.9.

p. 443 In the second margin note, Equation 4.11.29 should be 4.11.30.

p. 443 In Equation 4.11.30, right-hand side, it should be sup_k, not $\sup_R$.

p. 445 Clarification, not correction: We get π in the numerator in Equation 4.11.3 because

$$\int \frac{1}{1+u^2}\,du = \arctan u;$$

$$\lim_{u\to\pm\infty} \arctan u = \pm\frac{\pi}{2}.$$

p. 445, in the line after Equation 4.11.43, $1/2x$ should have been written $\frac{1}{2x}$ or $1/(2x)$.

p. 446, Theorem 4.11.16: in the line immediately before Equation 4.11.45, $(f \circ \varphi)|\det[D\Phi]|$ should be $(f \circ \Phi)|\det[\mathbf{D}\Phi]|$. In Equation 4.11.45 an absolute value sign is misplaced; the equation should be

$$\int_V f(\mathbf{v})|d^n\mathbf{v}| = \int_U (f \circ \Phi)(\mathbf{u}) \left|\det[\mathbf{D}\Phi(\mathbf{u})]\right| |d^n\mathbf{u}|.$$

p. 446 in Equation 4.11.46, the sum is over $\mathrm{vol}_n C$:

$$\sum_{\substack{C\in\mathcal{D}_N(\mathbb{R}^n),\\ \overline{C}\cap\partial U_R\neq\phi}} \mathrm{vol}_n C < \frac{1}{R},$$

p. 448: In Theorem 4.11.18, "except perhaps on a set of $\mathbf{x}$ of volume 0" should be "except perhaps for $\mathbf{x}$ in a set of volume 0." In Equation 4.11.59, it might be clearer to have the $|d^n\mathbf{x}|$ not part of the fractions:

$$\begin{aligned} DF(t) &= \lim_{h\to0}\frac{F(t+h)-F(t)}{h} = \lim_{h\to0}\int_{\mathbb{R}^n}\frac{f(t+h,\mathbf{x})-f(t,\mathbf{x})}{h}|d^n\mathbf{x}| \\ &= \int_{\mathbb{R}^n}\lim_{h\to0}\frac{f(t+h,\mathbf{x})-f(t,\mathbf{x})}{h}|d^n\mathbf{x}| = \int_{\mathbb{R}^n} D_t f(t,\mathbf{x})\,|d^n\mathbf{x}|. \end{aligned}$$

p. 452 Exercise 4.1.11 belongs with the exercises for Section 4.2. In addition, in the displayed equation of part (b), the (A) in the denominator should be $M(A)$.

p. 452 Exercise 4.1.12: "dilation of a function by a of a function" should be "dilation by a of a function." First line of Exercise 4.1.12 part (b): dyadic cubes, not canonical cubes

p. 453 Exercise 4.1.16 part (a) is badly stated and should be ignored.

p.453–454, Exercise 4.1.17, the equations after "Show that if $f : \mathbb{R}^n \to \mathbb{R}$ is integrable, then" should read as follows (upper and lower integrals, not upper and lower sums, so $\mathring{U}(f)$ not $\mathring{U}_N(f)$, and so on):

$$\begin{aligned} \mathring{U}(f) &= \lim_{N\to\infty}\sum_{C\in\mathcal{D}_N(\mathbb{R}^n)} M_{\mathring{C}}(f)\,\mathrm{vol}_n(C) \\ \mathring{L}(f) &= \lim_{N\to\infty}\sum_{C\in\mathcal{D}_N(\mathbb{R}^n)} m_{\mathring{C}}(f)\,\mathrm{vol}_n(C) \\ \overline{U}(f) &= \lim_{N\to\infty}\sum_{C\in\mathcal{D}_N(\mathbb{R}^n)} M_{\overline{C}}(f)\,\mathrm{vol}_n(C) \\ \overline{L}(f) &= \lim_{N\to\infty}\sum_{C\in\mathcal{D}_N(\mathbb{R}^n)} m_{\overline{C}}(f)\,\mathrm{vol}_n(C). \end{aligned}$$

p. 454 hint for Exercise 4.3.1: it is Proposition 4.3.6, not Theorem 4.3.6, and $y = \sqrt{1-x^2}$, not $y = \sqrt{x^2-1}$.

p. 454-455 Exercise 4.3.2, part (a) should start with the second line of page 455, i.e., "(a) Let $f : \mathbb{R}^n \to \mathbb{R}$ be an integrable function that is symmetric ... ". The displayed formula should be

$$\int_{Q^n_{a,b}} f(\mathbf{x})\,|d^n\mathbf{x}| = n! \int_{P^n_{a,b}} f(\mathbf{x})\,|d^n\mathbf{x}|.$$

p. 455 Exercise 4.4.2 should read "Show that $X \subset \mathbb{R}^n$ has measure 0 if and only if for any $\epsilon > 0$ there exists an infinite sequence of balls ... ". ($X \subset \mathbb{R}^n$, not $X \in \mathbb{R}^n$, and addition of "for any $\epsilon > 0$")

p. 457 Exercise 4.5.11: The proof of Theorem 3.3.9 has been relegated to Appendix A.6, so probably you don't recall it. The way the exercise is worded, you might wonder whether we mean to suggest that with a different proof, it is possible to relax the condition that the second partials be continuous. We do not; without that condition, the theorem is false.

p. 459, in Exercise 4.6.4, p is used with two different meanings, and in part (e), e^{-4} should be e^{-x}. In addition, part (d) can be solved using Student MATLAB only for values of m up to 4; for $m = 5$, the professional version of MATLAB is needed. In part (e), it would probably be clearer to say "for each of the five values of m above," not "for each of the degrees above."

The exercise should read (a) Find the equations that must be satisfied by points $x_1 < \cdots < x_m$ and weights $w_1 < \cdots < w_m$ so that the equation

$$\int_0^\infty p(x)e^{-x}dx = \sum_{k=1}^m w_k f(x_k)$$

is true for all polynomials p of degree $\leq d$.

(b) For what number d does this lead to as many equations as unknowns?

(c) Solve the system of equations when $m = 1$.

(d) Use Newton's method to solve the system for $m = 2, \ldots, 5$.

(e) For each of the five values of m above, approximate

$$\int_0^\infty e^{-x} \sin x\,dx \quad \text{and} \quad \int_0^\infty e^{-x} \ln x\,dx.$$

and compare the approximations with the exact values.

p. 460 In Exercise 4.6.5, a dx is missing; the equation should be

$$\int_{-\infty}^\infty p(x)e^{-x^2}\,dx = \sum_{i=0}^k w_i p(x_i)$$

In addition, note that while in Exercise 4.6.4 the integral was from 0 to ∞, here it is from $-\infty$ to ∞.

p. 461 In Exercise 4.8.1, "first column, not "first row." (Rows would work, but the text discusses computing determinants using development by the first column.)

p. 462, Exercise 4.8.6. part (b) should say column operations, not row operations.

p. 463 Exercise 4.9.1 should read

(a) Show that $\Delta(T) = |\det T|$ is the unique mapping $\Delta : \mathrm{Mat}\,(n,n) \to \mathbb{R}$ that satisfies

(1) For all $T \in \mathrm{Mat}\,(n,n)$, we have $\Delta(T) \geq 0$.

(2) The function Δ is a symmetric function of the columns (i.e., switching two columns does not change the value).

(3) For all $T = \big[\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n\big] \in \mathrm{Mat}\,(n,n)$, we have

$$\Delta\big[a\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n\big] = |a|\Delta\big[\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n\big].$$

(4) For all $T = \big[\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n\big] \in \mathrm{Mat}\,(n,n)$, we have

$$\Delta\big[\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n\big] = \Delta\big[\vec{\mathbf{v}}_1 + a\vec{\mathbf{v}}_2, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n\big].$$

(5) $\Delta(I_n) = 1$.

(b) Show that $T \mapsto \mathrm{vol}_n(T(Q))$ satisfies the properties that characterize Δ.

p. 463 Exercise 4.9.2 should read

Use "dissection" (as suggested in Figure 4.9.2) to prove Equation 4.9.12 when E is type 2.

p. 464, Exercise 4.9.6., there should be a plus sign after $2x_2$:

$$\{\mathbf{x} \in \mathbb{R}^n \mid x_i \geq 0 \text{ for all } i = 1, \dots, n \text{ and } x_1 + 2x_2 + \cdots + nx_n \leq n\}?$$

p. 465 Exercise 4.10.7: "What is the volume of the region $Q_A(\vec{\mathbf{x}}) \leq 1$" ($Q_A$, not Q). This exercise should be starred.

p.465, Exercise 4.10.10 really belongs with exercises for Section 5.2 as it requires the relaxed definition of a parametrization introduced there. Also in Exercise 4.10.10: $0 \leq r < \infty$, not $0 \leq r \leq \infty$.

p. 467 In Exercise 4.11.2, the first line should be "Let $a_n = \frac{(-1)^{n+1}}{n}$" (the exponent is $n+1$, not n.)

p. 467, Exercise 4.11.3, two R in displayed equation should be $\mathbb{R}$:

$$\lim_{k\to\infty} \lim_{R\to\infty} \int_{\mathbb{R}} [f_k(x)]_R \, dx \neq \lim_{R\to\infty} \lim_{k\to\infty} \int_{\mathbb{R}} [f_k(x)]_R \, dx.$$

p. 468, Exercise 4.11.5: the (a) should be deleted as there is no part (b).