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MATH 2940 (Dis. 204, 208, 216) – Quiz #3

Answer the following questions in the space provided. (There is a question on the back!)
Justify all answers.

Problem 1. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 9 \\ -3 & k \end{pmatrix}$. What value(s) of k, if any, will make $AB = BA$?

Solution.

$$AB = \begin{pmatrix} 2 & 3 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 9 \\ -3 & k \end{pmatrix} = \begin{pmatrix} -7 & 18+3k \\ 2 & -9-k \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & 9 \\ -3 & k \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} -7 & -6 \\ -6-k & -9-k \end{pmatrix}$$

Thus, in order for $AB = BA$, we must have that

$$\begin{pmatrix} -7 & 18+3k \\ 2 & -9-k \end{pmatrix} = \begin{pmatrix} -7 & -6 \\ -6-k & -9-k \end{pmatrix},$$

and in particular, $18 + 3k = -6$ and $2 = -6 - k$. It is easy to see that these equations are satisfied if and only if $k = -8$, and thus $k = -8$ is the (unique) value for which $AB = BA$. □

Problem 2. True or False: If A is an $m \times n$ matrix and $\mathbf{b}$ is *any* vector in $\mathbb{R}^m$, then the set of all solutions $\mathbf{x}$ to the equation $A\mathbf{x} = \mathbf{b}$ forms a subspace of $\mathbb{R}^n$. (If true, say why. If false, provide a counterexample.)

Solution. False. If $\mathbf{b} \neq \mathbf{0}$, then $A\mathbf{x} = \mathbf{b}$ will not have $\mathbf{0}$ as a solution (since $A\mathbf{0} = \mathbf{0} \neq \mathbf{b}$). Thus, the set of a solutions to $A\mathbf{x} = \mathbf{b}$ will not contain $\mathbf{0}$, showing that it is not a subspace. [Alternatively, $A\mathbf{x} = \mathbf{b}$ may have no solutions, and since the empty set certainly does not contain $\mathbf{0}$, it is also not a subspace.] □

Problem 3. Use the algorithm discussed in §2.2 of the text to determine if the matrix

$$A = \begin{pmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ -2 & -6 & 4 \end{pmatrix}$$

has an inverse A^{-1}, and if so, compute it.

Solution.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ -4 & -7 & 3 & 0 & 1 & 0 \\ -2 & -6 & 4 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \to R_3 + 2R_1]{R_2 \to R_2 + 4R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 4 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \to R_3 + 2R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 10 & 2 & 1 \end{array}\right)$$

The matrix on the left of the above augmented matrix has a row of zeros, showing that the reduced echelon form of A cannot be I_3, and thus, A is not invertible (i.e., A^{-1} does not exist). $\square$