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MATH 2940 (Dis. 204, 208, 216) – Quiz #5

Answer the following questions in the space provided. (There is a question on the back!)
Justify all answers.

Problem 1. If the null space of a 5×4 matrix A is 2-dimensional, what is the dimension of the row space of A?

Solution. By the Rank Theorem, $\text{rank}(A) + \text{dimNul}(A) = 4$, since A has 4 columns. Thus,

$$\text{rank}(A) = 4 - 2 = 2,$$

and since $\text{dimRow}(A) = \text{rank}(A)$, we have that

$$\text{dimRow}(A) = 2.$$

$\square$

Problem 2. Find the steady state vector $\mathbf{x}$ for the stochastic matrix $P = \begin{pmatrix} 0.4 & 0.8 \\ 0.6 & 0.2 \end{pmatrix}$.

Solution. A steady state vector $\mathbf{x}$ is a probability vector such that $P\mathbf{x} = \mathbf{x}$, or equivalently, $(P - I_2)\mathbf{x} = \mathbf{0}$, where $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

$$P - I_2 = \begin{pmatrix} -0.6 & 0.8 \\ 0.6 & -0.8 \end{pmatrix} \xrightarrow{R_2 \to R_2 + R_1} \begin{pmatrix} -3/5 & 4/5 \\ 0 & 0 \end{pmatrix} \xrightarrow{R_1 \to (-5/3)R_1} \begin{pmatrix} 1 & -4/3 \\ 0 & 0 \end{pmatrix}.$$

So a general solution to the homogeneous equation $(P - I_2)\mathbf{x} = \mathbf{0}$ is of the form $\begin{pmatrix} x \\ \frac{3}{4}x \end{pmatrix}$ for $x \in \mathbb{R}$. A particular solution is $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$, and so $\mathbf{x} = \begin{pmatrix} \frac{4}{7} \\ \frac{3}{7} \end{pmatrix}$ is the steady state vector. $\square$

Problem 3. Compute the following determinant (by any method):

$$\begin{aligned}
\begin{vmatrix} 5 & -2 & 4 \\ 0 & 3 & -5 \\ 2 & -4 & 7 \end{vmatrix} &= (-1)^{2+1}(0)\begin{vmatrix} -2 & 4 \\ -4 & 7 \end{vmatrix} + (-1)^{2+2}(3)\begin{vmatrix} 5 & 4 \\ 2 & 7 \end{vmatrix} + (-1)^{2+3}(-5)\begin{vmatrix} 5 & -2 \\ 2 & -4 \end{vmatrix} \\
&= 0 + 3(35-8) + 5(-20+4) \\
&= 81 - 80 \\
&= 1
\end{aligned}$$