Vector-valued functions (and a bit more)

Name: Solutions

Math 1920 - Sections 221 and 222 - TA: Itamar Oliveira

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Problems

(1) State the type of quadric surface and describe the trace obtained by intersecting with the given plane.

(a) $x^2 + (\frac{y}{4})^2 + z^2 = 1$, $y = 0$.
Solution: Ellipsoid. The required trace is a circle.

(b) $(\frac{x}{3})^2 + (\frac{y}{5})^2 - 5z^2 = 1$, $x = 0$.
Solution: Hyperboloid of one sheet. The required trace is a hyperbola.

(2) Find an equation of the form $r = f(\theta, z)$ in cylindrical coordinates for the following surfaces.

(a) $z = x + y$.

Solution: $r = \frac{z}{\cos\theta + \sin\theta}$.

(b) $z = 3xy$.

Solution: $r = \sqrt{\frac{2z}{3\sin 2\theta}}$.

(3) Find an equation of the form $\rho = f(\theta, \phi)$ in spherical coordinates for the following surfaces.

(a) $z^2 = 3(x^2 + y^2)$.
Solution: $\{\rho = 0 \text{ or } \phi = \frac{\pi}{6} \text{ or } \phi = \frac{5\pi}{6}\}$.

(b) $x^2 - y^2 = 4$.
Solution: $\rho = \frac{2}{\sin\phi\sqrt{\cos 2\theta}}$.

(4) Use sine and cosine to parametrize the intersection of the surfaces $x^2 + y^2 = 1$ and $z = 4x^2$.
Solution: $\mathbf{r}(t) = \langle \cos t, \sin t, 4\cos^2 t \rangle$

(5) Let $\mathbf{r}(t) = \langle 3\cos t, 5\sin t, 4\cos t \rangle$. Show that $\|\mathbf{r}(t)\|$ is constant and use this to conclude that $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are orthogonal.

(6) A fighter plane, which can shoot a laser beam straight ahead, travels along the path

$$\mathbf{r}(t) = \langle t - t^3, 12 - t^2, 3 - t \rangle.$$

Show that the pilot cannot hit any target on the x-axis.