VECTOR-VALUED FUNCTIONS AND LIMITS

NAME: SOLUTIONS

Math 1920 - Sections 221 and 222 - TA: Itamar Oliveira

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REVIEW

(1) The *domain* $\mathcal{D}$ of a function $f(x_1, \ldots, x_n)$ is the set of n-tuples $(a_1, \ldots, a_n)$ in $\mathbb{R}^n$ for which the function is defined. For example, the domain of

$$f(x_1, \ldots, x_n) = \frac{1}{\|(x_1, \ldots, x_n)\|}$$

is $\boxed{\mathbb{R}^n - \{(0, \ldots, 0)\}}$ (1). The *range* of f is the set of values taken by f.

(2) The *graph* of a real-valued function f is the subset of $\mathbb{R}^3$ of points $(a, b, f(a, b))$, for (a, b) in the domain of f.

(3) A *vertical trace* is obtained by intersecting the graph with a vertical plane $x = a$ or $y = b$.

(4) A *level curve* is a curve in the xy-plane defined by an equation $f(x, y) = c$.

(5) The *contour map* shows the level curves $f(x, y) = c$ for equally spaced values of c. The spacing m is called the *contour interval*.

(6) A *level surface* is a surface in the xyz-space defined by an equation $f(x, y, z) = c$. If f represents temperature, we call the level surfaces *isotherms*.

(7) The limit of a product $f(x, y) = g(x)h(y)$ is a product of limits:

$$\lim_{(x,y)\to(a,b)} f(x, y) = \left(\lim_{x\to a} g(x)\right)\left(\lim_{y\to b} h(y)\right).$$

(8) A function f of two variables is *continuous* at $P = (a, b)$ if

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b).$$

(9) To prove that a limit does not exist, it suffices to show that the limits obtained along two different paths are not equal.

Problems

(1) Sketch the contour map of $f(x, y) = x^2 + y^2$ with level curves $c = 0, 4, 8, 12$.

(2) Draw a contour map of $f(x, y) = xy$ with an appropriate contour interval, showing at least 4 curves.

(3) Let the temperature in 3-space be given by $T(x, y, z) = x^2 + y^2 - z^2$. Draw isotherms corresponding to temperatures $T = -2, 0, 2$.

(4) Let the temperature in 3-space be given by $T(x, y, z) = x^2 + y^2 - z$. Draw isotherms corresponding to temperatures $T = -1, 0, 1$.

(5) Let $f(x, y) = \frac{x^3+y^3}{xy^2}$. Does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? (Hint: set $y = mx$ and show that the result depends on m)

SOLUTION: No.

(6) Use any method to evaluate the limit or show that it does not exist

(a) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{\sqrt{x^2+y^2}}$.
SOLUTION: The limit is 0. Use polar coordinates and the Squeeze Theorem.

(b) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$.
SOLUTION: It does not exist. Set $y = mx$ and show that the result depends on m.

(c) $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$.
SOLUTION: Multiply and divide by the conjugate of the denominator. The limit is 2.