

## REVIEW

- (1) We say that a point  $P = (a, b)$  is a *critical point* of  $f(x, y)$  if  $f_x(a, b) = 0$  or does not exist, and  $f_y(a, b) = 0$  or does not exist.
- (2) The local minimum or maximum values of  $f$  occur at critical points.
- (3) The *discriminant* of  $f(x, y)$  at  $P = (a, b)$  is

$$D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}^2(a, b).$$

- (4) *Second Derivative Test*: If  $P = (a, b)$  is a critical point of  $f(x, y)$ , then

$$D(a, b) > 0, \quad f_{xx}(a, b) > 0 \Rightarrow f(a, b) \text{ is a local minimum}$$

$$D(a, b) > 0, \quad f_{xx}(a, b) < 0 \Rightarrow f(a, b) \text{ is a local maximum}$$

$$D(a, b) < 0 \Rightarrow \text{saddle point}$$

$$D(a, b) = 0 \Rightarrow \text{test inconclusive}$$

- (5) If  $f$  is continuous on a closed and bounded domain  $\mathcal{D}$ , then  $f$  takes on both a minimum and maximum value on  $\mathcal{D}$ . The extreme values occur either at critical points in the interior of  $\mathcal{D}$  or at points on the boundary of  $\mathcal{D}$ .

## PROBLEMS

- (1) Find the critical points of the following functions, then use the Second Derivative test to determine whether they are local minima, local maxima, or saddle points (or state that the test fails).

(a)  $f(x, y) = x^4 + y^4 - 4xy.$

(b)  $f(x, y) = x - y^2 - \ln(x + y).$

- (2) Find the maximum of  $f(x, y) = y^2 + xy - x^2$  on the square  $0 \leq x \leq 2, 0 \leq y \leq 2.$

- (3) Determine the global extreme values of the function on the given domain.

(a)  $f(x, y) = x^3 - 2y; \quad 0 \leq x \leq 1, 0 \leq y \leq 1.$

(c)  $f(x, y) = 2xy - x - y; \quad y \leq 4, y \geq x^2.$

(b)  $f(x, y) = x^3 + x^2y + 2y^2; \quad x, y \geq 0, x + y \leq 1.$

- (4) Find three positive numbers that sum to 150 with the largest possible product of the three.