Lagrange Multipliers and Integration

Name: Solutions

Math 1920 - Sections 221 and 222 - TA: Itamar Oliveira

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Problems

(1) Find the point in the first quadrant on the curve $y = x + x^{-1}$ closest to the origin.

Solution: $P = \left(\frac{1}{2^{\frac{1}{4}}}, 2^{\frac{1}{4}} + \frac{1}{2^{\frac{1}{4}}}\right)$.

(2) Use Lagrange Multipliers to find the dimensions (i.e. radius and height) of a cylindrical can with a bottom but not top, of fixed volume V with minimum surface area.

Solution: $r = h = \left(\frac{V}{\pi}\right)^{\frac{1}{3}}$.

(3) Evaluate the following integrals using Fubini's theorem.

(a) $\int_0^1 \int_0^1 y\sqrt{1+xy}\, dy\, dx$.

Solution: $\frac{16}{15}\sqrt{2} - \frac{14}{15}$.

(b) $\int_0^1 \int_0^1 xe^{xy} dx\, dy$.

Solution: $e - 2$.

(4) Compute the double integral over the domain $\mathcal{D}$ indicated

(a) $f(x, y) = x$; $0 \le x \le 1$, $1 \le y \le e^{x^2}$.

Solution: $(e-2)/2$.

(b) $f(x, y) = \sin x$; bounded by $x = 0$, $x = 1$, $y = \cos x$.

Solution: $1/2$.

(5) Find the volume of the region bounded by $z = 40 - 10y$, $z = 0$, $y = 0$, and $y = 4 - x^2$.

Solution: 256.

(6) Find the average height of the "ceiling" in Figure 2 defined by $z = y^2 \sin x$ for $0 \le x \le \pi$, $0 \le y \le 1$.

Solution: $\frac{2}{3\pi}$.

(7) Find the triple integral of the function z over the ramp in the picture below. Here, z is the height above the ground.

Solution: 2.

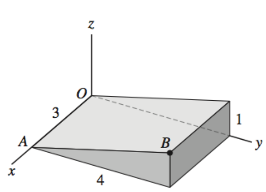

Figure 1: Problem 7

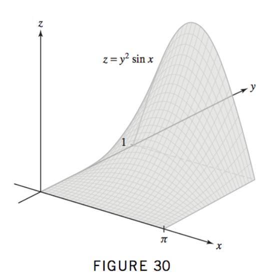

Figure 2: Problem 6