Pre-class activity (to be collected) Name: ____________

Math 1110 - Instructor: Itamar Oliveira due next class

Section 2.3 is about the precise definition of a limit. So far we have been working with limits from a more intuitive point of view, and we have been computing and operating with them even though we were not given a mathematical statement that says precisely what they are.

Like everything in math, things need to be formal and precise to be considered true. In the picture below we have an excerpt from the textbook that does the job. It may look hard at a first read, but the book does a great job at easing it: look at the subsection *Examples: testing the definition*, there is a nice picture explaining the $\epsilon - \delta$ (reads *epsilon-delta*) game. In this pre-class activity you will play the *Getting away from the devil in Malott Hall* game.

Background story: You are stuck in Malott Hall on a Sunday night with the devil and his only job is to be himself with you. There is only one problem: he needs to go back home to feed his cat. After thinking for a bit, he decides to let you go if you beat him at his game (keep in mind that he does not want to make it too hard so he can go home). If you win, you go. If you lose, he will feed you to his cat. Since you have homework due next Tuesday, you can't let that happen.

The game: He picked his favorite function $f(x) = x^{666} + 666$. You will take turns and the devil always goes first: he will tell you a number D (capital D, that we call *Devilpslon*) and you have to answer back with a number d (little d, that we call *devilta*) for which the following works:

$$\text{If } |x| < d, \quad \text{then } |f(x) - 666| < D.$$

After playing 10 rounds you realize the obvious: you will never win like that because he will just keep telling you different *Devilpslons* until you say a wrong *devilta.* To outsmart and force him to let you go, find a formula that exhibits a *devilta* d that works for any *Devilpslon* D that he tells you.

After beating the devil: Look at the following definition from the textbook and explain how it relates to the game. What did you just prove?

DEFINITION Let $f(x)$ be defined on an open interval about c, except possibly at c itself. We say that the **limit of $f(x)$ as x approaches c is the number L**, and write

$$\lim_{x \to c} f(x) = L,$$

if, for every number $\epsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all x,

$$0 < |x - c| < \delta \quad \Rightarrow \quad |f(x) - L| < \epsilon.$$