

# PRACTICE PROBLEMS

Math 1110 - Instructor: Itamar Oliveira

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**Important:** This list of problems is by no means intended to cover all kinds of questions that you can be asked on a quiz or on an exam. Also, it is **not** going to be collected, so you should focus on the problems involving the material that you are struggling with.

1. Find the domain and the range of the following functions:

(a)  $f(x) = \sqrt{16 - x^2}$ .

(c)  $f(x) = \tan(2x - \pi)$ .

(b)  $f(x) = \ln(x - 3) + 1$ .

(d)  $f(x) = \frac{\pi^{2020}}{\sin x}$ .

2. True or false? Justify.

(a) A vertical line intersects the graph of a function at most once.

(c) If  $f$  and  $g$  are even functions, then  $f \cdot g$  is even.

(b) A horizontal line intersects the graph of a function at most once.

(d) If  $f$  and  $g$  are odd functions, then  $f \cdot g$  is odd.

3. Sketch the graph of

$$g(x) = 1 + 3 \sin 2x.$$

Don't forget to label the axes and include an indication of scale.

4. Sketch the graph of the following function and use it to determine the values of  $a$  for which  $\lim_{x \rightarrow a} f(x)$  exists.

$$f(x) = \begin{cases} 1 + x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x < 1 \\ 2 - x & \text{if } x \geq 1 \end{cases}$$

5. Evaluate the limit, if it exists.

(a)  $\lim_{u \rightarrow 2} \frac{\sqrt{4u+1} - 3}{u - 2}$ .

(d)  $\lim_{x \rightarrow -2} \frac{x+2}{x^3+8}$ . *Hint: Factor  $a^3 + b^3$ .*

(b)  $\lim_{x \rightarrow -1} \frac{x^2 + 2x + 1}{u - 2}$ .

(e)  $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{\sqrt{x} - 1}$ . *Hint: Get rid of the radicals.*

(c)  $\lim_{t \rightarrow 0} \left( \frac{1}{t} - \frac{1}{t^2 + 1} \right)$ .

6. Compute

$$\lim_{x \rightarrow 0^+} \sqrt{x} \left( 1 + \sin^2 \left( \frac{2\pi}{x} \right) \right).$$

*Hint: Use the squeeze theorem.*

7. Compute the following limits, if they exist.

$$(a) \lim_{x \rightarrow 0} \frac{\tan 2x}{x}.$$

$$(b) \lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 4x}.$$

8. For what value of  $b$  is

$$g(x) = \begin{cases} \frac{x-b}{b+1} & \text{if } x < 0 \\ x^2 + b & \text{if } x > 0 \end{cases}$$

continuous at every  $x$ ?

9. Is there a number that is exactly 1 plus its cube?

10. Evaluate the limit, if it exists.

$$(a) \lim_{x \rightarrow \infty} \frac{x^4 + x^3}{12x^3 + e^{2020\pi}}.$$

$$(b) \lim_{x \rightarrow \infty} \frac{x^{2/3} + x^{-1}}{x^{2/3} + \cos^2 x}.$$

11. Find the equations for all horizontal asymptotes of

$$y = \frac{1 - x^2}{x^2 + 1}.$$

12. Find the equations for all vertical asymptotes of

$$y = \frac{x^2 + x - 6}{x^2 + 2x - 8}.$$

13. Using the definition, compute the derivative of the following function at the specified points:

$$r(s) = \sqrt{2s + 1}; \quad r'(0), r'(1).$$

14. Determine if the following piecewise defined function is differentiable at the origin:

$$g(x) = \begin{cases} x^{2/3} & \text{if } x \geq 0 \\ x^{1/3} & \text{if } x < 0 \end{cases}$$