Review on functions — Name: ____________

Math 1110 - Instructor: Itamar Oliveira — January 24, 2020

1 Basic concepts

Definition 1. A **function** f from a set D to a set Y is a rule that assigns a *unique* element $f(x) \in Y$ to each element $x \in D$.

(1) D is the ______ (1) of the function.

(2) The set of all output values of $f(x)$ as x varies throughout D is the ______ (2) of the function.

(3) The **graph** of f is ______ (3).

(4) Write down the example of a piecewise defined function you gave in your pre-class activity:

Definition 2. A function $y = f(x)$ is **even** if ______ (4). It is **odd** if ______ (5).

(1) Can a function be even and odd at the same time? ______ (6)

(2) Which functions from your pre-class activity are even? Which ones are odd?

Definition 3. A function p is a **polynomial** if

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots, a_n$ are real constants (called the **coefficients** of the polynomial).

(1) Which functions from your pre-class activity are polynomials?

Definition 4. A **rational function** is a quotient $f(x) = p(x)/q(x)$, where p and q are polynomials. The domain of a rational functions is the set of all real x for which $q(x) \neq 0$.

(1) Which functions from your pre-class activity are rational?

2 Shifting and scaling functions

(1) If $k > 0$ and $g(x) = f(x) + k$, how are the graphs of f and g related? What if $k < 0$?

(7)

(2) If $k > 0$ and $g(x) = f(x + k)$, how are the graphs of f and g related? What if $k < 0$?

(8)

(3) Look at Figure 1 and assign to each graph the corresponding function.

(a) $y = (x - 1)^2 - 4$. (9)

(b) $y = (x - 2)^2 + 2$. (10)

(c) $y = (x + 2)^2 + 2$. (11)

(d) $y = (x + 3)^2 - 2$. (12)

(4) Use Figure 2 (where all kinds of things are done to $y = \sqrt{x}$) as a reference to review what you read about scaling and reflecting graphs. For example, if $c > 1$, then $y = cf(x)$ stretches the graph of f vertically by a factor of c. Write down what happens to

(a) $c > 1$ and $y = \frac{1}{c} f(x)$. (13) .

(b) $c > 1$ and $y = f(cx)$. (14) .

(c) $c > 1$ and $y = f(x/c)$. (15) .

(d) $c = -1$ and $y = -f(x)$. (16) .

(e) $c = -1$ and $y = f(-x)$. (17) .

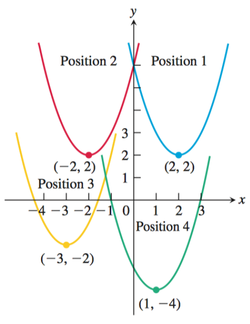

Figure 1: Shifting

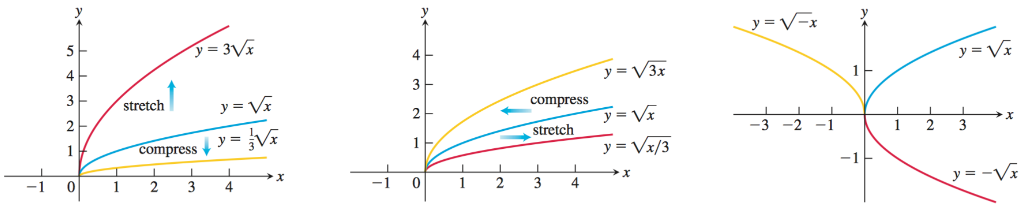

Figure 2: Scaling and reflecting

3 Trigonometric functions

1. Draw a right triangle and define the six basic trigonometric functions: sine, cosine, tangent, secant, cosecant and cotangent. If the hypotenuse of this triangle has length 1, what relation do you get between sine and cosine?

2. In the picture below, we have a rectangle and a right triangle "inside" it. Compute l_1 through l_6. What have you just proved (perhaps without noticing)?